The transmission of forced waves at a junction

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A thesis submitted in fulfillment of the requirements for the degree of
Master of Applied Science

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February 2013

Declaration

I declare that the work presented in this thesis is that of my own, except where due acknowledgment has been made, and has not been submitted previously, in whole or part, to qualify for any other academic award.

The content of the thesis is the result of work which has been carried out since 28th February, 2011, this being the official date of commencement of this programme.

Signature: ……………………..

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Date: February, 2013.

Acknowledgements

I wish to acknowledge the continued support and encouragement I have received from my project supervisor, Adjunct. Professor John Davy. Thanks for the many hours spent on my project, I was really impressed with your innovative ideas and boundless knowledge.

I would also like to thank my second supervisor Assoc Prof Philip Wilksch.

I must also thank Vladimir Pavasovic for his wonderful thesis Pavasovic (2006), a source of inspiration.

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List of Symbols

a	a constant
a_1	a constant
a_2	a constant
B	bending stiffness of the plate
B_1	bending stiffness of the plate in medium 1
B_2	bending stiffness of the plate in medium 2
B_0	the adiabatic bulk modulus of the air
c	the phase speed of the sound wave in a fluid medium
c_m	the speed of sound in the mth medium
c_i	freely propagating bending wave velocity of the ith plate
E(z)	variation of bending wave in z-axis direction
e	exponential function
exp	exponential function
f	frequency
f_c	critical frequency
F_x	reaction force per unit length at a boundary
I	intensity of the acoustic wave
I_i	incident intensity
I_r	reflected intensity
I_t	transmitted intensity
$I_{(i+r)}$	total intensity at $x=0$
$I_{tforced}$	forced transmitted intensity due to a forced incident wave
I_{tfree}	transmitted intensity of a normally incident freely propagating wave
j	is the square root of -1
k	wave number
k_a	airborne wave number
k_f	forced wave number
k_1	wave number in medium 1
k_2	wave number in medium 2
k_{tx}	transmitted wave number in the x direction

k_{ty}	transmitted wave number in the *y* direction
k_m	wave number in the *mth* medium
$\underline{k} = (k_x, k_y, 0)$	wave number vector
$\underline{k_t} = (k_{tx}, k_{ty}, 0)$	transmitted wave number vector
$\underline{k_r} = (k_{rx}, k_{ry}, 0)$	reflected wave number vector
$\underline{k_f} = (k_{fx}, k_{fy}, 0)$	forced incidence wave number vector
k_N	nearfield wavenumber
k_{N1}	nearfield wavenumber in medium 1
k_{N2}	nearfield wavenumber in medium 2
k_{xi}	incident wavenumber in *x*-axis direction
k_{xr}	reflected wavenumber in *x*-axis direction
k_{x2}	wavenumber in *x*-axis direction in medium 2
m	mass per unit area of plate
m_i	mass per unit area of the *ith* plate
M_{xz}	moment per unit length about the *z*-axis exerted on the beam cross sectional area normal to the *x*-axis
M_z	angular moment per unit length about the *z*-axis
p	acoustic pressure
p_f	forced acoustic pressure
p_r	reflected acoustic pressure
p_t	transmitted acoustic pressure
Q_x	shear force per unit length acting on a plane normal to the *x*-axis
r	ratio of forced incident wave no. to freely propagating wave no. in the incident media
r	amplitude of reflected wave
r	ratio of airborne wave no. to wave number in plate 1
r_j	amplitude of nearfield reflected wave
s^2	$\chi_a^2 \sin^2\theta_i = \sin^2\theta_1 = \kappa^2 \sin^2\theta_2$
t	time
t	amplitude of transmitted wave

t_j amplitude of transmitted nearfield wave

U volume velocity per unit volume inserted into the medium at position x

u_f forced complex particle velocity

u complex particle velocity

$\bar{u}$ complex conjugate of the particle velocity

u_x complex particle velocity in the x direction

u_{fx} forced complex particle velocity (in the direction of the x-axis)

u_{rx} reflected complex particle velocities (in the x-axis direction)

u_{tx} transmitted complex particle velocities (in the x-axis direction)

v_y transverse velocity of the plate in the direction of the y-axis

v_1 transverse bending wave in plate 1

v_2 transverse bending wave in plate 2

v_{1+} transverse velocity amplitude of a bending wave

w$_c$ angular critical frequency

w angular velocity

w$_z$ angular velocity of the plate about the z-axis

w$_{z2}$ angular velocity about the z-axis in plate 2

z impedance

Z_1 impedance in media 1

Z_2 impedance in media 2

Z_i impedance experienced by a freely propagating bending wave

θ_f incident angle

θ_r reflected angle

θ_t transmitted angle

η_1 damping loss factor of plate 1

ρ change in density from the ambient density

ρ_0 is the ambient density of the medium in which the wave is travelling

ρ_m ambient density of the mth medium

$\partial\Omega$ elemental strip of solid angle

< > average

$\overline{u}, \overline{P_f}, \overline{P_r}, \overline{Z_2}$ The bar over these symbols indicates taking the complex conjugate

Re real part of the imaginary number

κ k_2/k_1

φ angle to normal of wavenumber vector of exciting acoustic plane wave

χ_a ratio of forced incident wave number

ω angular frequency

Δ $\psi\left(j\sqrt{1-s^2}-\sqrt{1+s^2}\right)+\left(j\sqrt{\kappa^2-s^2}-\sqrt{\kappa^2+s^2}\right)$

ψ $\frac{B_2 k_2^2}{B_1 k_1^2}$

α $\frac{k_1}{k_2}$

β $\frac{\mathrm{Z}_1}{\mathrm{Z}_2}$

θ_u $\begin{cases} \frac{\pi}{2} & \text{if} \quad \kappa \geq \chi_a \\ \arcsin\left(\frac{\kappa}{\chi_a}\right) & \text{if} \quad \kappa < \chi_a \end{cases}$

Abstract

The European Cooperation in Science and Technology (COST) Action FP0702 “Net-Acoustics for Timber based Lightweight Buildings and Elements” is attempting to extend the EN12354 series of standards. The European Standards (EN) CEN (2000a), CEN (2000b), CEN (2000c), CEN (2000d), CEN (2000e), CEN (2000f) series, EN 12354 gives methods for predicting the propagation of sound and vibration in buildings. The first four parts of this series have also been published as the International Organization for Standardization (ISO) ISO15712 series of standards (ISO (2005a), ISO (2005b), ISO (2005c), ISO (2005d)). These series of standards are complemented by the ISO 10848 series of standards (ISO (2006a), ISO (2006b), ISO (2006)), which specific methods of measuring the flanking sound transmission in buildings.

This research has raised the question of whether the vibration transmission at a wall junction depends on how the wall is excited (by either a sound wave or a mechanical shaker). The aim of this research is to investigate this question and hence contribute to the revision of the series of standards. Villot and Guigou-Carter (2000) have considered this problem but their equations (10) and (13) appear to be in error. This research will endeavour to derive the correct equations and repeat Villot and Guigou-Carter’s calculations.

This thesis shows that the transmission of forced bending waves is different from the transmission of freely propagating bending waves. However, Villot and Guigou-Carter’s (2000) calculations, for a pinned junction between two panels that are the same, over estimates the difference. The case of a forced wave in a fluid incident on a plane interface surface where the properties of the fluid may change is investigated first. It is shown that the intensity propagated to and from the interface surface cannot be calculated separately for the forced incident wave and the freely propagating reflected wave, because the cross terms in the intensity calculation cannot be canceled. This implies that a transmission factor or coefficient cannot be calculated. This is the reason for the error in Villot and Guigou-Carter’s (2000) equation (10).

The calculations are then extended to the pinned junction between the two panels case, considered by Villot and Guigou-Carter. It is shown that their diffuse field weighting is also in error.

Chapter 1 Introduction

Flanking sound transmission is the transmission of sound from one room to another other than via the common wall of the two rooms (E. Gerretsen (1986), E. Gerretsen (1979), Nightingale (1995)). The current theory is only valid for heavy weight single leaf walls. Many researchers have addressed this area of research, see the papers by Villot (2002), Nightingale and Bosmans (2003), Davy, Mahn, Guigou-Carter, and Villot (2012); (Eddy Gerretsen, 2007), CSTC (2008), Mahn (2008), and Davy (2009). Researchers are able to measure the flanking transmission of many building systems (Brunskog and Chung (2011), Crispin, Ingelaere, Van Damme, and Wuyts (2006) and Guigou-Carter, Villot, and Roland (2006)). Lightweight building elements typically have critical frequencies in or above the frequency range of interest, so the current theory does not apply to them. The European Cooperation in Science and Technology (COST) Action FP0702 "Net-Acoustics for Timber based Lightweight Buildings and Elements" has been attempting to extend the EN12354 series of standards (ISO (2005a), ISO (2005b), ISO (2005c) and ISO (2005d)) for calculating the flanking sound transmission of single leaf heavy weight walls to lightweight walls and in particular to more complicated light weight walls. The research work of this action has raised the question of whether the transmission of vibration from one wall to another wall depends on whether the excited wall is excited mechanically by a shaker or acoustically by a sound field. The answering of this question will enable the extension of EN12354 to lightweight walls on a more rational basis. This extension is particularly important for Australia because Australia has much more lightweight construction than most parts of Europe.

Villot and Guigou-Carter (2000) have considered this problem but their equations (10) and (13) appear to be in error. They have published another paper on this subject in which their measurement methods have been reconsidered Guigou-Carter *et al*. (2006).

This research will endeavour to derive the correct equations and repeat Villot and Guigou-Carter's calculations.

Airborne excitation of a wall with a single frequency of sound at a single angle of incidence produces a forced bending wave in the wall with a wavelength which is equal to the trace wavelength of the incident sound on the wall. When this forced bending wave is reflected at the edges of the wall, it produces resonant bending waves and exponentially decaying nearfields. Excitation with a mechanical shaker produces resonant bending waves and exponentially decaying nearfields. This research will calculate theoretically the difference between the transmission of forced bending waves and resonant bending waves from an excited wall to a wall connected to the excited wall. For most walls, the transverse velocity of the resonant bending waves is larger than the transverse velocity of the forced bending waves. Villot and Guigou-Carter (2000) suggest that the transmission of forced bending waves through the junction can be ignored. Incident acoustic wave excitation and the mechanically excited excitation will be considered. Acoustic excitation induces forced and resonant bending waves. Excitation by a mechanical shaker induces only resonant bending waves. So by comparing the two cases of transmission of bending wave energy from one wall to another wall via a common junction, it can be determined if the forced bending wave transmission is significant or not. The aim is to find the velocity difference between the two walls, when the wall is excited acoustically and mechanically. Will the velocity difference be the same? It is suspected that it will be different because in the mechanical case there is only a resonant bending wave, while in the airborne case there is both a forced bending wave and resonant bending wave. The question is by how much will the velocity transmission differ?

The derivation of the relevant equations in Cremer, Heckl, and Petersson (2005) will be studied as will their extension to equation (9) of Villot and Guigou-Carter (2000). The difference between the transmission of forced and resonant waves will first be investigated for the simpler case of sound waves in a fluid medium. Then the correct versions of equations (10) and (13) of Villot and Guigou-Carter (2000) will be derived using the knowledge gained in the sound wave case. These corrected equations will then be used to predict the difference between forced bending wave transmission and resonant bending wave transmission from an excited wall to an attached wall.

Chapter 2 Derivation of the fluid media sound equations

2.1 Introduction

The aim of this thesis is to look at the transmission of forced bending waves at a junction of two plates. In this chapter the simpler case of forced waves in a fluid medium are considered because there are less variables and the equations are slightly simpler.

In this chapter, the case of the transmission of a forced sound wave from one infinite half space fluid medium to another is considered. First the case of normal incidence, when the two media are the same is considered.

It would not be possible to generate a forced wave in a three dimensional fluid medium because a distributed three dimensional volume velocity source would be needed. However the concept of a forced wave is theoretically possible and most derivations of the three dimensional fluid wave equation do include a distributed three dimensional volume velocity source. A point source is usually introduced by making the spatial distribution of this volume velocity source a spatial Dirac delta function.

Forced waves in a fluid medium, do exist practically in the two dimensional case of the air cavity in a double wall system when the width of the cavity is small compared to the wavelength of sound. They also exist practically in the one dimensional case of a microphone turbulence screen when the internal cross sectional dimensions of the microphone turbulence screen tube are small compared to the wavelength of sound.

2.2 Normal Incidence with the same media on both sides of the junction

The compressibility equation is

$$p = \frac{B_0}{\rho_0}\rho = c^2\rho, \tag{2.1}$$

where ρ is the change in density from the ambient density (density fluctuation due to the sound wave), p is the acoustic pressure from the ambient pressure. B_0 is the adiabatic bulk modulus of the air. B_0 is a measure of the volumetric incompressibility or volume stiffness of the medium. ρ_0 is the ambient density of the medium in which the wave is traveling. c is the phase speed of the sound wave in the gaseous or liquid medium. Because these media are non-dispersive, the group velocity (the velocity at which energy is transported) and the signal velocity are also equal to the phase velocity c.

$$c^2 = \frac{B_0}{\rho_0}. \tag{2.2}$$

Equation (2.2) relates the phase speed of the sound wave to the adiabatic bulk modulus and the density of the medium.

The continuity equation is

$$\frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial u}{\partial x} = \rho_0 U. \tag{2.3}$$

where u is the acoustic particle velocity of the sound wave in the positive x-axis direction. $\frac{\partial u}{\partial x}$ is the gradient in the x-axis direction of the acoustic particle velocity. The acoustic particle velocity is the velocity of the medium caused by the sound wave. U is the volume velocity per unit volume inserted into the medium at position x. Ambient refers to the climatic conditions of the medium before it is disturbed by the acoustic wave travelling through it.

The following two equations are Newton's inertia equations, where t is the time.

$$\rho_0 \frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = 0. \tag{2.4}$$

Differentiating (2.3) with respect to t gives

$$\frac{\partial^2 \rho}{\partial t^2} + \frac{\partial}{\partial t}(\rho_0 \frac{\partial u}{\partial x}) = \rho_0 \frac{\partial U}{\partial t}. \tag{2.5}$$

Differentiating (2.4) with respect to *x* gives

$$\frac{\partial}{\partial x}(\rho_0 \frac{\partial u}{\partial t}) + \frac{\partial^2 p}{\partial x^2} = 0. \tag{2.6}$$

Differentiating equation (2.1) twice with respect to *t* gives

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = \frac{\partial^2 \rho}{\partial t^2}. \tag{2.7}$$

Use equation (2.7) to replace the value of $\frac{\partial^2 \rho}{\partial t^2}$ in the following equation gives

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = -\rho_0 \frac{\partial U}{\partial t}. \tag{2.8}$$

This is the one dimensional wave equation. To solve the one dimensional wave equation for the forced pressure, requires that its variables (*p* and *U*) are defined, differentiated and then placed into equation (2.8). This is done below.

Let the volume velocity equal

$$U = U_f \exp[j(\omega t - k_f x)], \tag{2.9}$$

and the acoustic pressure equal

$$p = p_f \exp[j(\omega t - k_f x)], \tag{2.10}$$

where ω is the angular frequency.

Firstly *p* (equation (2.10)) will be differentiated twice with respect to *t* and *x*.

Differentiating *p* twice with respect to *x* gives

$$\frac{\partial^2 p}{\partial t^2} = j^2 \omega p = -\omega^2 p. \tag{2.11}$$

Next U is differentiated with respect to t.

$$\frac{\partial U}{\partial t} = j\omega U. \tag{2.12}$$

Substituting values for $\frac{\partial^2 p}{\partial x^2}$, $\frac{\partial^2 p}{\partial t^2}$(equation (2.11)), and $\frac{\partial U}{\partial t}$ (equation (2.12)) into the one dimensional wave equation ((2.8)) gives

$$(-k_f^2 + \frac{\omega^2}{c^2})p = -j\omega\rho_0 U. \tag{2.13}$$

The use of equation (2.9) and (2.10) to replace p and U in equation (2.13) gives

$$p_f = \frac{-j\omega\rho_0}{k^2 - k_f^2} U_f, \tag{2.14}$$

where

$$k = \frac{\omega}{c}, \tag{2.15}$$

is the wave number and where $\omega = 2\pi f$.

The boundary of the two half infinite fluid media in the plane x=0.

The forced solution of the non-homogeneous equation (2.8) is given by equation (2.14) where the forcing only occurs in medium 1 where x<0. Thus $U_f = 0$ if x>0 and

$$P_f = \begin{cases} \frac{-j\omega\rho_0}{k^2 - k_f^2} U_f & x < 0 \\ 0 & x > 0 \end{cases}. \tag{2.16}$$

2.3 Media same for *x<0* and *x>0*, normal incidence forced wave

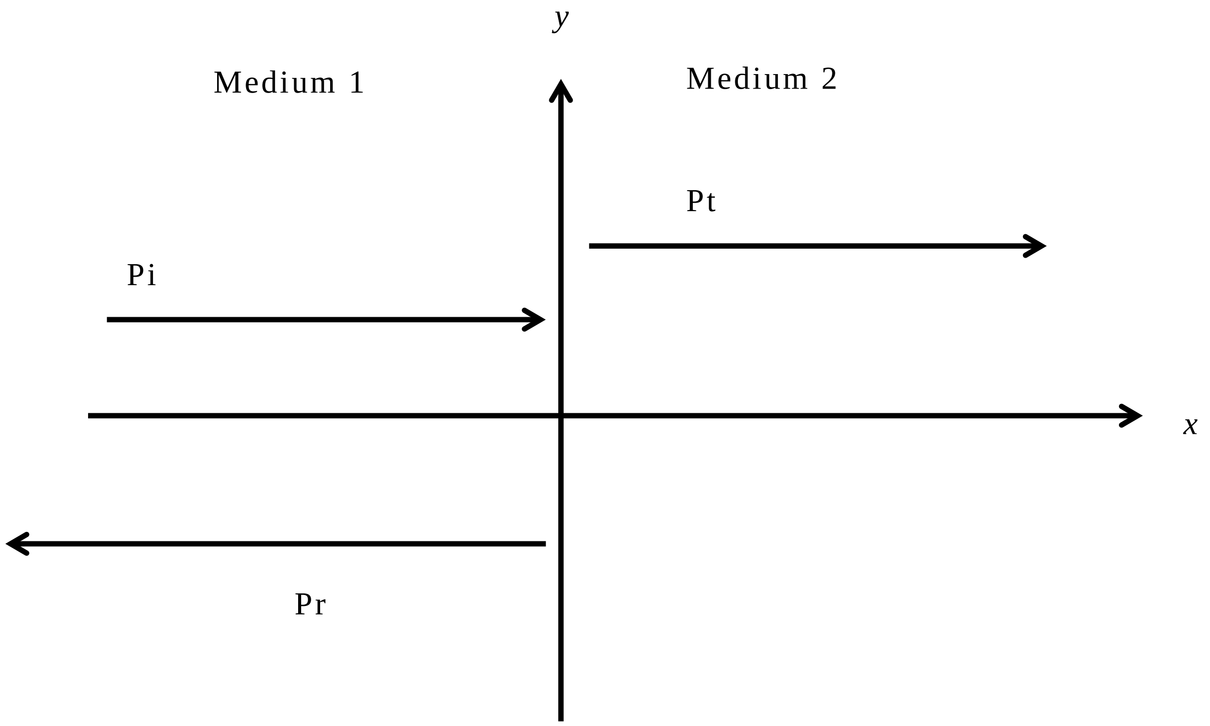

Figure 2.1 The two dimensional case.

In this section the solution to the homogenous version of equation (2.8) will be calculated. The reflected and transmitted pressures will also be calculated. Finally the incident transmitted intensity will be calculated.

To obtain the continuity of pressure and acoustic particle velocity at x=0, solutions of the homogeneous version of equation (2.8) need to be added. The homogeneous version of equation (2.8) is

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0. \tag{2.17}$$

To solve the above equation the pressure will be defined and differentiated with respect to x and t. Then these solutions will be replaced into equation (2.17).

To begin the pressure is defined to be

$$p = a\exp[j(\omega t - mx)]. \tag{2.18}$$

Equation (2.18) is differentiated twice with respect to x and t and the results are put into equation (2.17) to give

$$-m^2 a + \frac{\omega^2}{c^2}a = 0. \tag{2.19}$$

If a is not equal to zero then equation (2.19) may be rearranged to give

$$m = \pm\frac{\omega}{c} = \pm k. \tag{2.20}$$

The result of equation (2.20) is put into equation (2.18) to give the solution. Thus the general solution of the homogeneous equation is

$$p = a_1 e^{[j(\omega t - kx)]} + a_2 e^{[j(\omega t + kx)]}. \tag{2.21}$$

Hence the solution for the case of the only incident wave being a forced incident wave in the first medium is

$$p = \begin{cases} p_f \exp[j(\omega t - k_f x)] + p_r \exp\left[j(\omega t + kx)\right] & x < 0 \\ & . \\ p_t \exp[j(\omega t - kx)] & x > 0 \end{cases} \tag{2.22}$$

Thus,

$$\rho_0 u = \begin{cases} \dfrac{k_f p_f}{\omega} \exp[j(\omega t - k_f x)] - \dfrac{k p_r}{\omega} \exp[j(\omega t + kx)] & x \leq 0 \\ & . \\ \dfrac{k}{\omega} p_t \exp[j(\omega t - kx)] & x \geq 0 \end{cases} \tag{2.23}$$

At $x=0$ and $t=0$ with the same medium on both sides, the continuity of pressure gives

$$p = p_f + p_r = p_t. \tag{2.24}$$

Using equation (2.23), the continuity of the particle velocity at $x=0$ and $t=0$ gives

$$u = \frac{k_f}{\rho_0 \omega} p_f - \frac{k}{\rho_0 \omega} p_r = \frac{k}{\rho_0 \omega} p_t. \tag{2.25}$$

To find the transmitted pressure the above equation is multiplied by $\rho_0 \omega$

$$k_f p_f - k p_r = k p_t. \tag{2.26}$$

the transmitted pressure is

$$p_t = \frac{k + k_f}{2k} p_f. \tag{2.27}$$

Rearranging this gives

$$p_r = \frac{k_f - k}{2k} p_f. \tag{2.28}$$

The total intensity propagated in the positive x direction if $x<0$ is

$$I_{(i+r)} = \mathrm{Re}(p.\bar{u}) \,. \tag{2.29}$$

$I_{(i+r)}$ is the total intensity, Re is the real part of the complex number, p is the pressure and $\bar{u}$ is the complex particle velocity. The total pressure on the $x<0$ side is

$$p = p_f \exp[j(\omega t - k_f x)] + p_r \exp[j(\omega t + kx)]. \tag{2.30}$$

From equation (2.25) the complex particle velocity is given by

$$\bar{u} = \frac{k_f}{\rho_0 \omega}\overline{p_f} \exp[-j(\omega t - k_f x)] - \frac{k}{\rho_0 \omega}\overline{p_r} \exp[-j(\omega t + kx)]. \tag{2.31}$$

The bar over the u in equation (2.31) indicates the operation of taking the complex conjugate (changing the sign of the imaginary part of the complex number). This is also sometimes shown with a superscript *.

Putting equation (2.30) and (2.31) into equation (2.29) gives

$$\begin{aligned} I_{(i+r)} &= \mathrm{Re}(p.\bar{u}) \\ &= \mathrm{Re}\left\{\begin{array}{l} \left(p_f \exp[j(\omega t - k_f x)] + p_r \exp[j(\omega t + kx)]\right)\times \\ \left(\dfrac{k_f}{\rho_0 \omega}\overline{p_f} \exp[-j(\omega t - k_f x)] - \dfrac{k}{\rho_0 \omega}\overline{p_r} \exp[-j(\omega t + kx)]\right) \end{array}\right\}. \end{aligned} \tag{2.32}$$

this becomes

$$I_{(i+r)} = \mathrm{Re}\left(\frac{1}{\rho_0 \omega}\left[\begin{array}{l} k_f |p_f|^2 - \dfrac{k(k_f - k)^2}{4k^2}|p_f|^2 \\ -\dfrac{k(k_f - k)}{2k}|p_f|^2 \exp[-j(k_f + k)x] \\ +k_f \dfrac{(k_f - k)}{2k}|p_f|^2 \exp[j(k_f + k)x] \end{array}\right]\right). \tag{2.33}$$

Thus the total intensity has been calculated.

2.4 The total intensity when *x*=0 (at the junction of the two media)

In this section the total intensity is calculated at the junction of the plates (*x=0*).

Equation (2.33) is considered again. Equation (2.33) can be written as

$$I_{(i+r)}=\frac{\left|p_f\right|^2}{\rho_0\omega}\left[\frac{4k^2k_f}{4k^2}-\frac{k(k_f-k)^2}{4k^2}-\frac{2kk(k_f-k)}{2k2k}+k_f\,\frac{2k(k_f-k)}{2k2k}\right]. \tag{2.34}$$

Thus

$$I_{(i+r)}=\frac{\left|p_f\right|^2}{\rho_0\omega}\left[\frac{k^2+k_f^2+2kk_f}{4k}\right]. \tag{2.35}$$

Using the following relationships

$$\begin{aligned}&k=\frac{\omega}{c},\\&\omega=kc,\\&c=\frac{\omega}{k},\end{aligned} \tag{2.36}$$

gives

$$I_{(i+r)}=\frac{\left|p_f\right|^2}{\rho_0 c}\frac{(k+k_f)^2}{4k^2}, \tag{2.37}$$

and thus the total intensity at the junction of the two media is

$$I_{(i+r)}=\frac{\left|p_f\right|^2}{\rho_0 c}\left(\frac{k+k_f}{2k}\right)^2. \tag{2.38}$$

2.5 The transmitted intensity ($I_t(x)$) when $x>0$

In this section the transmitted intensity is calculated.

The transmitted intensity is given by

$$I_t = \mathrm{Re}(p.\overline{u}). \tag{2.39}$$

The pressure is given by

$$p = p_t \exp[j(\omega t - kx)]. \tag{2.40}$$

The complex conjugate of the complex particle velocity is given by

$$\overline{u} = \frac{k}{\rho_0 \omega} \overline{p_t} \exp[-j(\omega t - kx)]. \tag{2.41}$$

Now equation (2.40) and (2.41) are put into equation (2.39). This gives

$$I_t = p_t \exp[j(\omega t - kx)] \frac{k}{\rho_0 \omega} \overline{p_t} \exp[-j(\omega t - kx)]. \tag{2.42}$$

Equation (2.42) is simplified down to

$$I_t = |p_t|^2 \frac{k}{\rho_0 \omega}. \tag{2.43}$$

From equation (2.27)

$$p_t = \frac{k + k_f}{2k} p_f. \tag{2.44}$$

$$I_t = \frac{|p_f|^2}{\rho_0 c} \left(\frac{k + k_f}{2k} \right)^2. \tag{2.45}$$

2.6 The intensity carried by the reflected wave (x<0) if propagating alone.

In this section the reflected intensity is calculated.

The intensity of the reflected wave in the x<0 region is

$$I_r = (\overline{p.u}). \tag{2.46}$$

The value used for p is the same as equation (2.30).

$$p = p_f \exp[j(\omega t - k_f x)] + p_r \exp\left[j(\omega t + kx)\right]. \quad (2.47)$$

The value used for the complex particle velocity $\overline{u}$ is the same as equation (2.31)

$$\overline{u} = \frac{k_f}{\rho_0 \omega} \overline{p_f} \exp[-j(\omega t - k_f x)] - \frac{k}{\rho_0 \omega} \overline{p_r} \exp[-j(\omega t + kx)]. \quad (2.48)$$

The reflected intensity (I_r) is the same as the incident intensity (I_i)

$$I_r = \mathrm{Re}\begin{bmatrix} \dfrac{k_f |p_f|^2}{\rho_0 \omega} - \dfrac{k|p_r|^2}{\rho_0 \omega} - \dfrac{k}{\rho_0 \omega} p_f \overline{p_r} \exp[-j(k_f + k)x] \\ + \dfrac{k_f}{\rho_0 \omega} \overline{p_f} p_r \exp[+j(k_f + k)x] \end{bmatrix}. \quad (2.49)$$

In this case the incident forced wave is not being considered, thus $p_f = 0$

$$I_r = -\frac{k|p_r|^2}{\rho_0 \omega}. \quad (2.50)$$

$$|p_r|^2 = \frac{(k_f - k)^2}{4k^2} |p_f|^2. \quad (2.51)$$

Using the relationship in equation (2.36) it is found that

$$I_r = -\frac{|p_f|^2}{\rho_0 c} \left(\frac{k - k_f}{2k} \right)^2. \quad (2.52)$$

2.7 Media different for *x<0* and *x>0*. A normally incident forced wave is firstly considered. The transmitted intensity and the incident and reflected intensity is found.

In this section the media are different on both sides of the junction. The complex particle velocity is calculated. The reflected and transmitted pressures are calculated.

Then the transmitted intensity is calculated. Finally the incident intensity is calculated.

Note that I_r is negative because the reflected intensity is propagating in the negative x-axis direction rather than in the positive x-axis direction. If ρ_1 and c_1 apply for x<0 and ρ_2 and c_2 apply for x>0, then we define the impedances of the two media to be

$$Z_1 = \rho_1 c_1, \text{ and } Z_2 = \rho_2 c_2 \tag{2.53}$$

and the wave numbers to be

$$k_1 = \frac{\omega}{c_1} \text{ and } k_2 = \frac{\omega}{c_2} \tag{2.54}$$

The pressure in the two media is given by

$$p = \begin{cases} p_f \exp\left[j(\omega t - k_f x)\right] + p_r \exp[j(\omega t + k_1 x)] & x < 0 \\ p_t \exp[j(\omega t - k_2 x)] & x > 0 \end{cases}. \tag{2.55}$$

After much substitution,

$$u = \begin{cases} \frac{k_f p_f}{\rho_1 \omega} \exp[j(\omega t - k_f x)] - \frac{k_1 p_r}{\rho_1 \omega} \exp[j(\omega t + k_1 x)] & x < 0 \\ \frac{k_2 p_t}{\rho_2 \omega} \exp[j(\omega t - k_2 x)] & x > 0 \end{cases}. \tag{2.56}$$

To find the transmitted pressure, equation (2.24) is considered as it applies at $x=0$ and $t=0$. Equation (2.24) is

$$p = p_f + p_r = p_t \tag{2.57}$$

Also using the version of equation (2.25) with the appropriate wave numbers gives

$$u = \frac{k_f}{\rho_1 \omega} p_f - \frac{k_1}{\rho_1 \omega} p_r = \frac{k_2}{\rho_2 \omega} p_t. \tag{2.58}$$

The complex particle velocity can now be expressed as

$$u = \frac{k_f}{\rho_1 k_1 c_1} p_f - \frac{k_1}{\rho_1 k_1 c_1} p_r = \frac{k_2}{\rho_2 k_2 c_2} p_t. \qquad (2.59)$$

The transmitted pressure is

$$p_t = \frac{(k_1 + k_f)}{k_1} \frac{Z_2}{Z_1 + Z_2} p_f. \qquad (2.60)$$

To find the reflected pressure,

$$\frac{k_f}{k_1} \frac{p_f}{Z_1} - \frac{P_r}{Z_1} = \frac{p_t}{Z_2}. \qquad (2.61)$$

Rearranging this equation gives the reflected pressure as

$$p_r = \frac{(k_f Z_2 - k_1 Z_1)}{k_1 (Z_2 + Z_1)} p_f. \qquad (2.62)$$

Transmitted intensity at $x=0$ is

$$I_t = \mathrm{Re}(p.\bar{u}). \qquad (2.63)$$

gives

$$I_t = \mathrm{Re}\left(\frac{|p_t|^2}{\overline{Z_2}} \exp[j(\omega t - k_2 x)] \exp[-j(\omega t - k_2 x)] \right). \qquad (2.64)$$

Therefore equation (2.64) reduces down to

$$I_t = \mathrm{Re}\left(\frac{|p_t|^2}{\overline{Z_2}} \right). \qquad (2.65)$$

From equation (2.60)

$$|p_t|^2 = \left(\frac{(k_1 + k_f)}{k_1} \right)^2 \frac{{Z_2}^2}{(Z_1 + Z_2)^2} |p_f|^2. \qquad (2.66)$$

The transmitted intensity is found to be

$$I_t = \left(\frac{(k_1 + k_f)}{k_1}\right)^2 \frac{Z_2}{(Z_1 + Z_2)^2} \left|p_f\right|^2. \tag{2.67}$$

Total intensity at x=0 is

$$I_{(i+r)} = \mathrm{Re}(p.\overline{u}). \tag{2.68}$$

$$I_{(i+r)} = \mathrm{Re}\begin{pmatrix} \frac{k_f}{k_1 Z_1}\left|p_f\right|^2 - \frac{\overline{p_r} p_f}{Z_1}\exp[-j(k_1 + k_f)x] \\ + \frac{k_f}{k_1}\frac{\overline{p_f} p_r}{Z_1}\exp[j(k_1 + k_f)x] - \frac{\left|p_r\right|^2}{Z_1} \end{pmatrix}. \tag{2.69}$$

At x=0

$$I_{(i+r)} = \mathrm{Re}\left(\frac{k_f}{k_1 Z_1}\left|p_f\right|^2 - \frac{\overline{p_r} p_f}{Z_1} + \frac{k_f}{k_1}\frac{\overline{p_f} p_r}{Z_1} - \frac{\left|p_r\right|^2}{Z_1}\right). \tag{2.70}$$

From equation (2.62)

$$p_r = \frac{(k_f Z_2 - k_1 Z_1)}{k_1 (Z_2 + Z_1)} p_f. \tag{2.71}$$

$$I_{(i+r)} = \left(\frac{k_1 + k_f}{k_1}\right)^2 \frac{Z_2}{(Z_2 + Z_1)^2}\left|p_f\right|^2. \tag{2.72}$$

Hence this is the simplest form of the total intensity.

2.8 The acoustic particle velocity in the oblique incidence case.

In this section the complex particle velocity in the x direction is calculated.

The three dimensional momentum equation is

$$\nabla p = -\rho_{m0} \frac{\partial \mathrm{u}}{\partial t} \ , \tag{2.73}$$

where u is the vectoral acoustic particle velocity and ρ_{m0} is the ambient density of the *m*th medium. The *x* component of equation (2.74) is

$$\frac{\partial p}{\partial x} = -\rho_{m0} \frac{\partial u_x}{\partial t}. \tag{2.74}$$

The complex particle velocity in the *x* direction is given by

$$u_x = u_{x0} e^{j\omega t}. \tag{2.75}$$

Differentiating with respect to t gives

$$\frac{\partial u_x}{\partial t} = j\omega u_x. \tag{2.76}$$

Putting equation (2.76) into (2.74) gives

$$\frac{\partial p}{\partial x} = -j\omega \rho_{m0} u_x. \tag{2.77}$$

gives

$$\frac{\partial p}{\partial x} = -jk_m \rho_{m0} c_m u_x. \tag{2.78}$$

The pressure is given by

$$p = p_0 e^{-j(k_x x + k_y y)} e^{j\omega t}. \tag{2.79}$$

Thus the complex particle velocity in the *x* direction is

$$u_x = \frac{1}{Z_m} \frac{k_x}{k_m} p. \tag{2.80}$$

where Z_m can be Z_1 or Z_2, k_x can be k_{1x}, k_{rx}, or k_{fx} and will either be real or imaginary

and k_m can be k_1 or k_2, *r* denotes the reflected wave and *f* the incident forced wave.

2.9 Proof of no power flow in a nearfield

In this section the complex particle velocity is calculated. Then it is proved that there is no power flow in a nearfield.

When total internal reflection occurs, a non-propagating nearfield is produced in the second medium. There is no power flow with a nearfield. This is proved below for a transmitted nearfield wave. Consider

$$\frac{\partial p}{\partial x} = -\rho_0 \frac{\partial u}{\partial t}. \tag{2.81}$$

The sound pressure in a nearfield is given by

$$p = p_t e^{j\omega t} e^{-k_{tx} x} e^{-jk_{ty} y}. \tag{2.82}$$

$$-\frac{\partial p}{\partial x} \frac{1}{\rho_0} = \frac{\partial u}{\partial t}. \tag{2.83}$$

$$\frac{-k_{tx}}{-\rho_0} p_t e^{j\omega t} e^{-k_{tx} x} e^{-jk_{ty} y} = \frac{\partial u}{\partial t}. \tag{2.84}$$

Equation (2.179) is integrated with respect to *t* to give the complex particle velocity

Equation (2.181) becomes

$$u = \frac{-jk_{tx}}{\omega \rho_0} p_t e^{j\omega t} e^{-k_{tx} x} e^{-jk_{ty} y}. \tag{2.85}$$

Taking the complex conjugate of the particle velocity gives

$$\overline{u} = \frac{+jk_{tx}}{\omega \rho_0} \overline{p_t} e^{-j\omega t} e^{-k_{tx} x} e^{+jk_{ty} y}. \tag{2.86}$$

The intensity is given by

$$I = \mathrm{Re}(p.\overline{u}). \tag{2.87}$$

Equation (2.176) and (2.184) gives

$$p.\overline{u} = (p_t e^{j\omega t} e^{-k_{tx}x} e^{-jk_{ty}y}).(\frac{+jk_{tx}}{\omega\rho_0}\overline{p_t} e^{-j\omega t} e^{-k_{tx}x} e^{+jk_{ty}y}). \tag{2.88}$$

After some simplification this becomes

$$p.\overline{u} = \frac{+jk_{tx}}{\omega\rho_0}|p_t|^2 e^{-2k_{tx}x}. \tag{2.89}$$

The real part of equation (2.187) is zero because it is purely imaginary. Hence

$$I = \mathrm{Re}(p.\overline{u}) = 0 \tag{2.90}$$

Thus there is no power flow for a nearfield.

2.10 Derivation of transmitted and reflected pressures for oblique incidence.

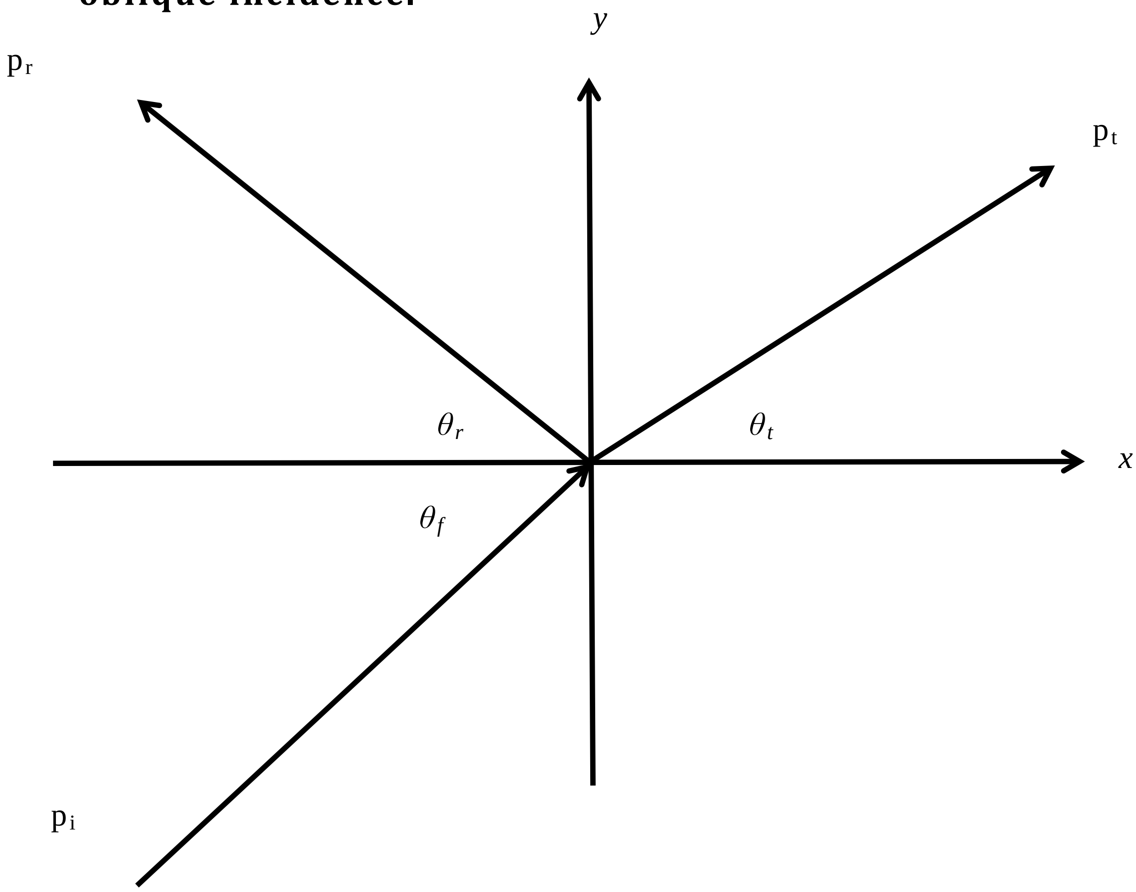

Figure 2.2: A figure showing the co-ordinate figures used in this section.

In this section the sines and cosines of the transmitted angle, the reflected angle and the transmitted angle are calculated. The transmitted wave number in the *x* and *y* direction is calculated. The reflected wave number in the *x* and *y* direction is calculated. The forced wave number in the *x* and *y* direction is calculated. The transmitted pressure is calculated. Finally the reflected pressure is calculated.

The plane wave number **k**= $(k_x, k_y, 0)$ lies in the *x y* plane and makes an angle of θ with the *x*-axis. The position variable is $\underline{\mathbf{x}}$=(*x*, *y*, *z*). The time is *t* and ω is the angular velocity. Θ is the angle between **k** and **x**. The axes have to be rotated so that there is no component of wave number in the *z*-axis direction. The plane acoustic wave varies as

$$\begin{aligned} & e^{j(\omega t - |\underline{k}||\underline{x}|\cos\theta)} = e^{j(\omega t - \underline{k}.\underline{x})} \\ & = e^{j(\omega t - (k_x x + k_y y))}, \end{aligned} \tag{2.91}$$

The component of the forced wave number in the *y* axis direction is

$$k_y = k_f \sin\theta_f, \tag{2.92}$$

where k_f is the magnitude of the forced wave number and θ_f is the incident angle. In the *y* axis direction the forced incident (*f*), reflected (*r*), and transmitted wave numbers must all be the same because of the continuity of the acoustic pressure and particle velocity

$$k_f \sin\theta_f = k_r \sin\theta_r = k_t \sin\theta_t. \tag{2.93}$$

Because the reflected wave is freely propagating, the reflected wave number is equal to the wave number in medium one, where the wave phase speed is c_1

$$k_r = k_1 = \frac{\omega}{c_1}. \tag{2.94}$$

Because the transmitted wave is freely propagating, the transmitted wave number (k_t) is equal to the wave number (k_2) in the second medium where the wave phase speed is c_2

$$k_t = k_2 = \frac{\omega}{c_2}. \tag{2.95}$$

the continuity of acoustic pressure and particle velocity is given by

$$\frac{k_f}{k_1}\frac{\omega}{c_1}\sin\theta_f = \frac{\omega}{c_1}\sin\theta_r = \frac{\omega}{c_2}\sin\theta_t, \tag{2.96}$$

The wave phase speeds in mediums one and two are given by

$$c_1 = \frac{\omega}{k_1}, \quad \text{and} \quad c_2 = \frac{\omega}{k_2}. \tag{2.97}$$

Solving for $\sin\theta_t$ gives

$$\begin{aligned}\sin\theta_t &= \frac{c_2}{c_1}\frac{k_f}{k_1}\sin\theta_f = \frac{k_f}{k_1}\frac{\omega}{k_2}\frac{k_1}{\omega}\sin\theta_f \\ &= \frac{k_f}{k_2}\sin\theta_f.\end{aligned} \tag{2.98}$$

Solving for $\sin\theta_r$ gives

$$\sin\theta_r = \frac{k_f}{k_1}\sin\theta_f. \tag{2.99}$$

From equation (2.80), the components of the forced incidence, reflected and transmitted complex particle velocities (in the direction of the x-axis) are given by

$$u_{fx} = \frac{k_f x p_f}{k_1 Z_1} = \frac{k_f p_f \cos\theta_f}{k_1 Z_1}, \tag{2.100}$$

and

$$u_{rx} = \frac{k_{rx} p_r}{k_1 Z_1} = \frac{p_r \cos\theta_r}{Z_1}, \tag{2.101}$$

and

$$u_{tx} = \frac{k_{tx} p_t}{k_2 Z_2} = \frac{p_t \cos\theta_t}{Z_2}. \tag{2.102}$$

The forced, reflected and transmitted pressures are given by

$$p_f + p_r = p_t. \tag{2.103}$$

The components of the forced, reflected and transmitted complex particle velocities in the direction of the x-axis are related by

$$u_f \cos\theta_f - u_r \cos\theta_r = u_t \cos\theta_t, \tag{2.104}$$

because the acoustic particle velocity is continuous at the junction of the two media (x=0). Equation (2.104) can be written using equations (2.100), (2.101), (2.102) as

$$\frac{k_f p_f \cos\theta_f}{k_1 Z_1} - \frac{p_r \cos\theta_r}{Z_1} = \frac{p_t \cos\theta_t}{Z_2}. \tag{2.105}$$

From the mathematical identity,

$$\cos^2\theta + \sin^2\theta = 1, \tag{2.106}$$

There follows

$$\cos\theta_f = \sqrt{1-\sin^2\theta_f}. \tag{2.107}$$

From equation (2.99),

$$\cos\theta_r = \sqrt{1-\sin^2\theta_r} = \sqrt{1-\frac{k_f^2}{k_1^2}\sin^2\theta_f}. \tag{2.108}$$

From equations (2.107) and (2.98) $\cos\theta_t$ is found to be

$$\begin{aligned}\cos\theta_t &= \sqrt{1-\sin^2\theta_t} = \sqrt{1-\frac{k_f^2}{k_2^2}\sin^2\theta_f} \\ &= \sqrt{1-\frac{k_1^2}{k_2^2}\frac{k_f^2}{k_1^2}\sin^2\theta_f}.\end{aligned} \tag{2.109}$$

The wave number that is transmitted through the junction at a particular angle to the x and y axis is

$$\begin{aligned}\mathrm{k}_t &= \left(k_{tx}, k_{ty}\right) = \left(\sqrt{k_2^2 - k_f^2\sin^2\theta_f}, k_f\sin\theta_f\right) \\ &= \left(k_2\sqrt{1-\frac{k_1^2}{k_2^2}\frac{k_f^2}{k_1^2}\sin^2\theta_f}, k_f\sin\theta_f\right),\end{aligned} \tag{2.110}$$

and the wave number that is reflected from the junction at a particular angle to the x and y axis is

$$\underline{k_r} = (k_{rx}, k_{ry}) = \left(\sqrt{k_1^2 - k_f^2 \sin^2\theta_f}, k_f \sin\theta_f\right)$$
$$= \left(k_1\sqrt{1 - \frac{k_f^2}{k_1^2}\sin^2\theta_f}, k_f \sin\theta_f\right), \tag{2.111}$$

and the wave number that is forced onto the junction of the two media at a particular angle is

$$\underline{k_f} = (k_{fx}, k_{fy}) = \left(\sqrt{k_f^2 - k_f^2 \sin^2\theta_f}, k_f \sin\theta_f\right)$$
$$= \left(k_f\sqrt{1 - \sin^2\theta_f}, k_1 \frac{k_f}{k_1}\sin\theta_f\right). \tag{2.112}$$

The z-axis component has been omitted in the above equations because it is always zero. Because the acoustic pressure is continuous at the junction of the two media (x=0),

$$p_f + p_r = p_t. \tag{2.113}$$

Equation (2.105) is

$$\frac{k_f \cos\theta_f}{k_1 Z_1} p_f - \frac{\cos\theta_r}{Z_1} p_r = \frac{\cos\theta_t}{Z_2} p_t \,. \tag{2.114}$$

Multiplying equation (2.113) by $\frac{\cos\theta_r}{Z_1}$ and adding equation (2.114) gives

$$\left[\frac{k_f \cos\theta_f}{k_1 Z_1} + \frac{\cos\theta_r}{Z_1}\right] p_f = \left[\frac{\cos\theta_t}{Z_2} + \frac{\cos\theta_r}{Z_1}\right] p_t. \tag{2.115}$$

Thus the transmitted pressure amplitude is

$$p_t = \left[\frac{k_f \cos\theta_f Z_2 + k_1 Z_2 \cos\theta_r}{k_1 Z_1 \cos\theta_t + k_1 Z_2 \cos\theta_r}\right] p_f. \tag{2.116}$$

Multiplying equation (2.113) by $\frac{\cos\theta_t}{Z_2}$ and subtracting it from equation (2.114) gives

$$\left[\frac{k_f \cos\theta_f}{k_1 Z_1} - \frac{\cos\theta_t}{Z_2}\right] p_f = \left[\frac{\cos\theta_r}{Z_1} + \frac{\cos\theta_t}{Z_2}\right] p_r . \tag{2.117}$$

Thus the reflected pressure is

$$p_r = \frac{\lfloor k_f \cos\theta_f Z_2 - k_1 Z_1 \cos\theta_t \rfloor}{\left[k_1 Z_2 \cos\theta_r + k_1 Z_1 \cos\theta_t\right]} p_f . \tag{2.118}$$

2.11 Derivation of total and forced intensity for oblique incidence. Different Media.

In this section the total intensity in the first medium is calculated. The transmitted intensity in the second medium is then calculated. The total and transmitted intensity are found to be the same.

The incident intensity in the direction of the x-axis is given by

$$I_{(i+r)x} = \mathrm{Re}(u_x . \overline{p}), \tag{2.119}$$

where u_x is the particle velocity in the x direction, and $\overline{p}$ is the complex conjugate of the pressure.

Thus

$$I_{(i+r)x} = \mathrm{Re}\left[(u_{fx} - u_{rx})(\overline{p_f} + \overline{p_r})\right] , \tag{2.120}$$

The forced particle velocity is

$$u_f = \frac{k_f}{k_1 Z_1} p_f , \tag{2.121}$$

and the forced particle velocity in the x direction is

$$u_{fx} = \frac{k_{fx}}{k_1 Z_1} p_f. \tag{2.122}$$

The reflected particle velocity in the x direction is

$$u_{rx} = \frac{k_{rx}}{k_1 Z_1} p_r. \tag{2.123}$$

Thus the intensity is

$$\begin{aligned} I_i &= \mathrm{Re}\left(\frac{k_{fx}}{k_1 Z_1} p_f \overline{p_f} + \frac{k_{fx}}{k_1 Z_1} p_f \overline{p_r} - \frac{k_{rx}}{k_1 Z_1} p_r \overline{p_f} - \frac{k_{rx}}{k_1 Z_1} p_r \overline{p_r} \right) \\ &= \frac{\left| p_f \right|^2 k_{fx}}{Z_1 k_1} \mathrm{Re}\left[\left\{ 1 + \overline{\left(\frac{p_r}{p_f} \right)} \right\} \left\{ 1 - \frac{k_{rx}}{k_{fx}} \frac{p_r}{p_f} \right\} \right]. \end{aligned} \tag{2.124}$$

Equation (2.124) can be further simplified. The term $1 - \frac{k_{rx}}{k_{fx}} \frac{p_r}{p_f}$ will now be considered

From equation (2.118)

$$\frac{p_r}{p_f} = \frac{k_{fx} Z_2 - \frac{k_1}{k_2} k_{tx} Z_1}{k_{rx} Z_2 + \frac{k_1}{k_2} k_{tx} Z_1}. \tag{2.125}$$

Hence

$$\frac{k_{rx}}{k_{fx}} \frac{p_r}{p_f} = \frac{k_{rx} Z_2 - \frac{k_{rx}}{k_{fx}} \frac{k_1}{k_2} k_{tx} Z_1}{k_{rx} Z_2 + \frac{k_1}{k_2} k_{tx} Z_1}. \tag{2.126}$$

Thus

$$\begin{aligned} &1 - \frac{k_{rx}}{k_{fx}} \frac{p_r}{p_f} \\ &= \frac{(1 + \frac{k_{rx}}{k_{fx}}) \frac{k_1}{k_2} k_{tx} Z_1}{k_{rx} Z_2 + \frac{k_1}{k_2} k_{tx} Z_1}. \end{aligned} \tag{2.127}$$

Evaluating $1+\overline{\left(\frac{p_r}{p_f}\right)}$ gives

$$\overline{\left(\frac{p_r}{p_f}\right)}=\frac{k_{fx}Z_2-\frac{k_1}{k_2}\overline{k_{tx}}Z_1}{\overline{k_{rx}}Z_2+\frac{k_1}{k_2}\overline{k_{tx}}Z_1}, \tag{2.128}$$

because k_{fx}, Z_2, k_1, k_2, Z_1 are all real numbers and the complex conjugate of a real number is the real number.

The component of the reflected wave number in the x-axis direction

$$k_{rx}=k_1\sqrt{1-\frac{k_f^2}{k_1^2}\sin^2\theta_f}\ , \tag{2.129}$$

can be real or imaginary because $\frac{k_f^2}{k_1^2}\sin^2\theta_f$ can be greater than 1, making the numerical value under the square root sign negative. In this case the square root number is an imaginary number.

The component of the transmitted wave number in the x-axis direction

$$k_{tx}=k_2\sqrt{1-\frac{k_f^2}{k_2^2}\sin^2\theta_f}\ , \tag{2.130}$$

can be real or imaginary, because the value under the square root sign can be negative, making the square root an imaginary number.

Thus

$$1+\overline{\left(\frac{p_r}{p_f}\right)}=\frac{\left(k_{fx}+\overline{k_{rx}}\right)Z_2}{\overline{k_{rx}}Z_2+\frac{k_1}{k_2}\overline{k_{tx}}Z_1}. \tag{2.131}$$

Hence putting equations (2.127) and (2.131) together gives

$$\left\{1+\left(\overline{\frac{p_r}{p_f}}\right)\right\}\left\{1-\frac{k_{rx}}{k_{fx}}\frac{p_r}{p_f}\right\}$$

$$=\frac{Z_2}{\left|k_{rx}Z_2+\frac{k_1}{k_2}k_{tx}Z_1\right|^2}(k_{fx}+\overline{k_{rx}})((1+\frac{k_{rx}}{k_{fx}})\frac{k_1}{k_2}k_{tx}Z_1). \tag{2.132}$$

Therefore the total intensity in the first medium in the x-axis direction is

$$I_{(i+r)x}=\frac{|p_f|^2 k_{fx}}{Z_1 k_1}\mathrm{Re}\left[\left\{\frac{\left(k_{fx}+\overline{k_{rx}}\right)Z_2}{\overline{k_{rx}}Z_2+\frac{k_1}{k_2}\overline{k_{tx}}Z_1}\right\}\left\{\frac{(1+\frac{k_{rx}}{k_{fx}})\frac{k_1}{k_2}k_{tx}Z_1}{k_{rx}Z_2+\frac{k_1}{k_2}k_{tx}Z_1}\right\}\right]$$

$$=\frac{|p_f|^2 Z_2|k_{fx}+k_{rx}|^2}{\left|k_{rx}Z_2+\frac{k_1}{k_2}k_{tx}Z_1\right|^2 k_2}\mathrm{Re}\left(k_{tx}\right). \tag{2.133}$$

The transmitted intensity in the second medium in the x-axis direction is given by

$$I_{tx}=\mathrm{Re}(u_{tx}\overline{p_t}). \tag{2.134}$$

The particle velocity transmitted in the x direction is given by

$$u_{tx}=\frac{k_{tx}}{k_2 Z_2}p_t. \tag{2.135}$$

Thus

$$I_{tx}=\mathrm{Re}(\frac{k_{tx}}{Z_2 k_2}|p_t|^2)$$

$$=\frac{|p_t|^2}{Z_2 k_2}\mathrm{Re}(k_{tx}). \tag{2.136}$$

From equation (2.116) the transmitted sound pressure is

$$p_t=\frac{(k_{fx}+k_{rx})Z_2}{\frac{k_1}{k_2}k_{tx}Z_1+k_{rx}Z_2}p_f. \tag{2.137}$$

Thus the modulus squared of the transmitted sound pressure is

$$|p_t|^2 = \frac{|k_{fx} + k_{rx}|^2 Z_2^2}{\left|\frac{k_1}{k_2} k_{tx} Z_1 + k_{rx} Z_2\right|^2} |p_f|^2. \tag{2.138}$$

Thus the transmitted intensity can be written as

$$\begin{aligned} I_{tx} &= \frac{|p_f|^2}{Z_2 k_2} \mathrm{Re}(k_{tx}) \frac{|k_{fx} + k_{rx}|^2 Z_2^2}{\left|k_{rx} Z_2 + \frac{k_1}{k_2} k_{tx} Z_1\right|^2} \\ &= \frac{|p_f|^2 Z_2 |k_{fx} + k_{rx}|^2}{\left|k_{rx} Z_2 + \frac{k_1}{k_2} k_{tx} Z_1\right|^2 k_2} \mathrm{Re}(k_{tx}). \end{aligned} \tag{2.139}$$

This is the same as the incident intensity given by equation (2.133).

2.12 Diffuse field incidence. Media the same. Analytical calculation of intensity for *r* less than or equal to 1.

In this section the forced transmitted intensity for diffuse field incidence is calculated and then normalized. The average transmitted intensity over the area of a hemisphere of solid angle 2π is calculated. The average forced transmitted intensity is calculated and then normalized. The three integrals in this equation are then evaluated using Gradshteyn and Ryzhik (1980).

The diffuse field incidence case (when the media are the same) is now considered. The letter r is defined to be the ratio of the incident forced wave number (k_f), to the freely propagating wave number (k).

$$r = \frac{k_f}{k}. \tag{2.140}$$

Because the media are the same

$$k_1 = k_2 = k, \tag{2.141}$$

$$\mathrm{Z}_1 = Z_2, \tag{2.142}$$

and

$$\theta_{\mathrm{r}} = \theta_t = \theta. \tag{2.143}$$

From equation (2.139) , the forced transmitted intensity due to a forced incident wave is

$$\begin{aligned} I_{tforced} &= \frac{\left|p_f\right|^2 Z \left|k_f \cos\theta_i + k\cos\theta\right|^2}{\left|kZ\cos\theta + kZ\cos\theta\right|^2} \frac{\mathrm{Re}(k\cos\theta)}{k} \\ &= \frac{\left|p_f\right|^2}{Z} \frac{\left|\frac{k_f}{k}\cos\theta_i + \cos\theta\right|^2}{\left|2\cos\theta\right|^2} \mathrm{Re}(\cos\theta). \end{aligned} \tag{2.144}$$

From the equality of the y axis components of the wave numbers

$$k_f \sin\theta_i = k\sin\theta, \tag{2.145}$$

and

$$\sin\theta = \frac{k_f}{k}\sin\theta_i = r\sin\theta_i. \tag{2.146}$$

The cosine functions can be easily seen to be

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-r^2\sin^2\theta_i}, \tag{2.147}$$

and

$$\cos\theta_i = \sqrt{1-\sin^2\theta_i}. \tag{2.148}$$

Thus the forced transmitted intensity is

$$I_{tforced} = \frac{\left|p_f\right|^2}{Z} \frac{\left|r\cos\theta_i + \sqrt{1-r^2\sin^2\theta_i}\right|^2}{\left|4(1-r^2\sin^2\theta_i)\right|} \mathrm{Re}(\sqrt{1-r^2\sin^2\theta_i}), \tag{2.149}$$

$$I_{tforced} = \begin{cases} 0 & \text{if } \left|\sin\theta_{\mathrm{i}}\right| \geq \dfrac{1}{r} \\ \dfrac{\left|p_f\right|^2}{Z} \dfrac{\left|r\cos\theta_i + \sqrt{1-r^2\sin^2\theta_i}\right|^2}{4\sqrt{(1-r^2\sin^2\theta_i)}} . & \text{if } \left|\sin\theta_{\mathrm{i}}\right| < \dfrac{1}{r} \end{cases} \tag{2.150}$$

The transmitted intensity of a normally incident ($\theta=0$) freely propagating wave ($r=1$) will be used to normalize the forced transmitted intensity.

$$I_{tfree}(\theta_i = 0) = \frac{|p_f|^2}{Z} \ . \tag{2.151}$$

$$\frac{I_{tforced}}{I_{tfree}(\theta_i = 0)} = \begin{cases} 0 & \text{if } |\sin\theta_{\mathrm{i}}| \geq \frac{1}{r} \\ \frac{(r\cos\theta_i + \sqrt{1 - r^2\sin^2\theta_i})^2}{4\sqrt{(1 - r^2\sin^2\theta_i)}} \ . & \text{if } |\sin\theta_{\mathrm{i}}| < \frac{1}{r} \end{cases} \tag{2.152}$$

The elemental strip of solid angle between θ_i and $\theta_i + d\theta_i$ is given by

$$\partial\Omega = 2\pi\sin\theta_i d\theta_i. \tag{2.153}$$

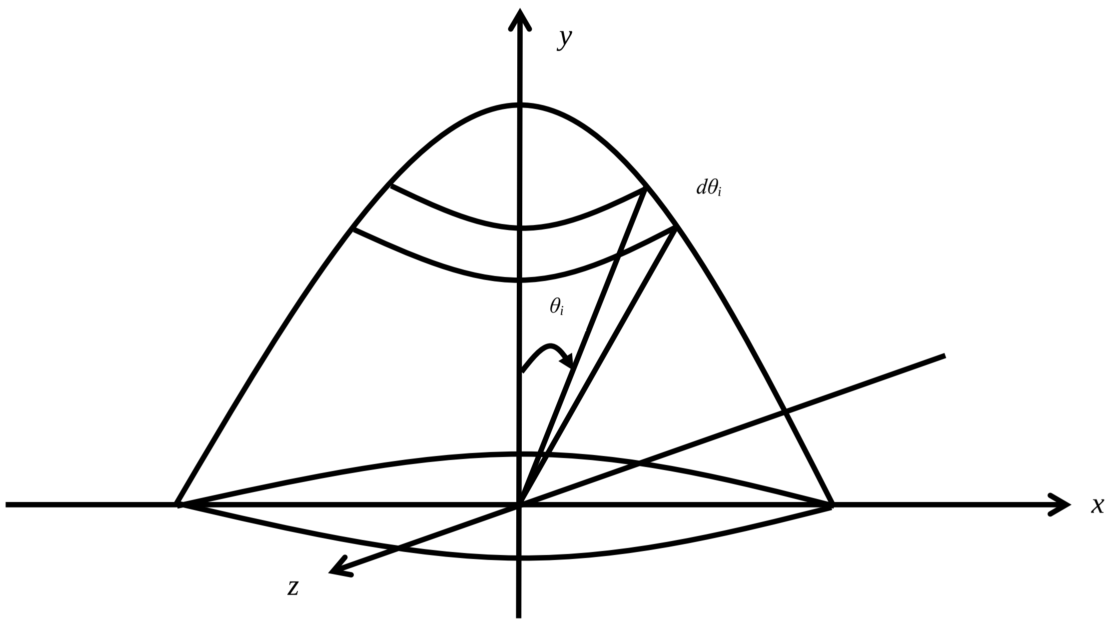

Figure 2.3 Graphical diagram of equation (2.258).

Thus the sum of the transmitted intensity over all angles of incidence is

$$2\pi \int_0^{\pi/2} I_t(\theta_i)\sin\theta_i d\theta_i. \tag{2.154}$$

The area of a hemisphere of a solid angle is 2π. Divide by 2π to obtain the average transmitted intensity over the solid angle.

$$\left\langle I_{tforced} \right\rangle = \frac{2\pi \int_0^{2\pi} I_t(\theta_i) \sin\theta_i d\theta_i}{2\pi}. \tag{2.155}$$

From equation (2.150) the transmitted intensity due to a freely propagating incident wave is

$$I_{tfree} = \frac{\left| p_f \right|^2}{Z} \cos\theta_i. \tag{2.156}$$

The average value over all angles of incidence is

$$(I_{tfree}) = \frac{\left| p_f \right|^2}{Z} \int_0^{\pi/2} \cos\theta_i \sin\theta_i d\theta_i = \frac{\left| p_f \right|^2}{2Z}. \tag{2.157}$$

The case when the ratio, r, of the forced incident wave number to the freely propagating wave number is now considered. From equations (2.150) and (2.155), the average of the transmitted intensity due to a forced incident wave is

$$Average(I_{tforced}) = \frac{\left| p_f \right|^2}{Z} \int_0^{\frac{\pi}{2}} \frac{(r\cos\theta_i + \sqrt{1 - r^2 \sin^2\theta_i})^2}{4\sqrt{1 - r^2 \sin^2\theta_i}} \sin\theta_i d\theta_i. \tag{2.158}$$

Put

$$x = \cos\theta_i\ , \tag{2.159}$$

then

$$dx = -\sin\theta_i d\theta, \tag{2.160}$$

$$\sin^2\theta_i = 1 - \cos^2\theta_i = 1 - x^2, \tag{2.161}$$

$$\begin{aligned} 1 - r^2 \sin^2\theta_i &= 1 - r^2(1 - x^2) \\ &= 1 - r^2 + r^2 x^2. \end{aligned} \tag{2.162}$$

The integral limits change to

$$\cos(0) = 1, \tag{2.163}$$

and

$$\cos(\frac{\pi}{2}) = 0. \tag{2.164}$$

Swapping the integrals limits because of the $-sin\theta$ converts the minus sign to a plus sign. The average transmitted intensity is normalized by dividing by the average transmitted intensity for a freely propagating incident wave.

$$\begin{aligned}\frac{(I_{tforced})}{(I_{tfree})} &= \frac{\left|p_f\right|^2}{Z}\frac{2Z}{\left|p_f\right|^2}\int_0^1 \frac{(rx+\sqrt{(1-r^2)+r^2x^2})^2}{\sqrt{(1-r^2)+r^2x^2}}dx \\ &= \frac{1}{2}\int_0^1 \frac{(rx+\sqrt{(1-r^2)+r^2x^2})^2}{\sqrt{(1-r^2)+r^2x^2}}dx.\end{aligned} \tag{2.165}$$

Expanding the top line of the integrand gives

$$\begin{aligned}&(rx+\sqrt{(1-r^2)+r^2x^2})^2 \\ &= r^2x^2 + 2rx\sqrt{(1-r^2)+r^2x^2} + ((1-r^2)+r^2x^2).\end{aligned} \tag{2.166}$$

Thus

$$\frac{\langle I_{tforced}\rangle}{\langle I_{tfree}\rangle} = \frac{r^2}{2}\int_0^1 \frac{x^2}{\sqrt{(1-r^2)+r^2x^2}} + \frac{2r}{2}\int_0^1 x\,dx + \frac{1}{2}\int_0^1 \sqrt{(1-r^2)+r^2x^2}dx. \tag{2.167}$$

These three integrals will be evaluated individually.

Put

$$R = a + bx + cx^2, \tag{2.168}$$

where

$$a = (1-r^2), \qquad b = 0, \qquad c = r^2 \tag{2.169}$$

and

$$\Delta = 4ac = 4(1-r^2)r^2. \tag{2.170}$$

The first integral

$$\frac{r^2}{2}\int_0^1 \frac{x^2}{\sqrt{(1-r^2)+r^2x^2}}, \tag{2.171}$$

Can be evaluate using integral number 2.264.3 on page 83 of Gradshteyn and Ryzhik (1980)

$$\int \frac{x^2 dx}{\sqrt{R}} = (\frac{x}{2c} - \frac{3b}{4c^2})\sqrt{R} + (\frac{3b^2}{8c^2} - \frac{a}{2c})\int \frac{dx}{\sqrt{R}}$$
$$= (\frac{x}{2r^2} - 0)\sqrt{(1-r)+r^2x^2} + (0 - \frac{(1-r^2)}{2r^2})\int \frac{dx}{\sqrt{R}}. \tag{2.172}$$

From the integral 2.261 in Gradshteyn and Ryzhik (1980).

$$\int \frac{dx}{\sqrt{R}} = \frac{1}{\sqrt{c}}\ln(2\sqrt{cR} + 2cx + b) = \frac{1}{\sqrt{r^2}}\ln(2\sqrt{r^2(1-r^2)+r^2x^2} + 2r^2x)$$
$$= \frac{x}{2r^2}\sqrt{(1-r^2)+r^2x^2} - \frac{(1-r^2)}{2r^3}\ln(2r\sqrt{(1-r^2)+r^2x^2} + 2r^2x). \tag{2.173}$$

Putting equation (2.173) into equation (2.172) gives

$$\int \frac{x^2}{\sqrt{(1-r^2)+r^2x^2}}dx$$
$$= \frac{x}{2r^2}\sqrt{(1-r^2)+r^2x^2} - \frac{(1-r^2)}{2r^3}\ln(2r\sqrt{(1-r^2)+r^2x^2} + 2r^2x). \tag{2.174}$$

Evaluating this integral between zero and one.

$$\int_0^1 \frac{x^2}{\sqrt{(1-r)^2 + r^2x^2}}dx$$
$$= \frac{1}{2r^2}\left[1 + \frac{(1-r^2)}{r}\ln(\frac{\sqrt{(1-r^2)}}{(1+r)})\right]. \tag{2.175}$$

The second integral is easily evaluated.

$$\int_0^1 xdx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}. \tag{2.176}$$

The third integral is evaluated using integral number 2.262.1 from Gradshteyn and Ryzhik (1980).

$$\int\sqrt{(1\text{-}\mathrm{r}^2)+r^2x^2}\,dx=\frac{(2cx+b)\sqrt{R}}{4c}+\frac{\Delta}{8c}\int\frac{dx}{\sqrt{R}}$$
$$=\frac{(2r^2x)\sqrt{(1-r^2)+r^2x^2}}{4r^2}+\frac{4(1-r^2)r^2}{8r^2}\int\frac{dx}{\sqrt{(1-r^2)+r^2x^2}}. \tag{2.177}$$

Now

$$\int\frac{dx}{\sqrt{(1-r^2)+r^2x^2}}=\frac{1}{r}\ln(2r\sqrt{(1-r^2)+r^2x^2}+2xr^2)$$
$$=\frac{1}{2}x\sqrt{(1-r^2)+r^2x^2}+\frac{1`}{2}\frac{(1-r^2)}{r}\ln(2r\sqrt{(1-r^2)+r^2x^2}+2xr^2). \tag{2.178}$$

Thus

$$\int_0^1 dx=\frac{1}{2}\sqrt{(1-r^2)+r^2}+\frac{1}{2}\frac{(1-r^2)}{r}\ln(2r\sqrt{(1-r^2)+r^2}+2r^2)$$
$$-\frac{1}{2}\frac{(1-r^2)}{r}\ln(2r\sqrt{(1-r^2)})$$
$$=\frac{1}{2}\left[1+\frac{(1-r^2)}{r}\ln(\frac{2r\sqrt{1-r^2+r^2}+2r^2}{2r\sqrt{1-r^2}})\right]=\frac{1}{2}\left[1+\frac{(1-r^2)}{r}\ln(\frac{1+r}{\sqrt{1-r^2}})\right]. \tag{2.179}$$

Putting the three integrals together gives

$$\frac{\langle I_{tforced}\rangle}{\langle I_{tfree}\rangle}=\frac{\mathrm{r}^2}{2}\int_0^1\frac{x^2}{\sqrt{(1-r^2)+r^2x^2}}+\frac{2r}{2}\int_0^1 xdx+\frac{1}{2}\int_0^1\sqrt{(1-r^2)+r^2x^2}\,dx$$
$$=\frac{r^2}{2}(\frac{1}{2r^2}\left[1+\frac{(1-r^2)}{r}\ln(\frac{\sqrt{(1-r^2)}}{(1+r)})\right]+\frac{r}{2}+\frac{1}{2}\frac{1}{2}\left[1+\frac{(1-r^2)}{r}\ln(\frac{(1+r)}{\sqrt{(1-r^2)}})\right]$$
$$=\frac{1}{4}\left[1+\frac{(1-r^2)}{r}\ln(\frac{\sqrt{(1-r^2)}}{(1+r)}\right]+\frac{r}{2}+\frac{1}{4}\left[1-\frac{(1-r^2)}{r}\ln(\frac{\sqrt{(1-r^2)}}{(1+r)}\right]$$
$$=\frac{1+r}{2}. \tag{2.180}$$

2.13 Diffuse field incidence. Media the same. *r* greater than or equal to 1.

In this section equation (2.167) is evaluated again but with different limits on the integral. From equation (2.150), if r is greater than or equal to one, the integral only needs to be evaluate over values of x for which $\left|\sin\theta_i\right| = \sqrt{1-x^2}$ is less than or equal to $1/r$. Thus the integral only needs to be evaluated for values when $\sqrt{1-x^2} < 1/r.$ This implies that

$$\begin{aligned} &1-x^2 < 1/r, \\ &x^2 > 1-1/r, \\ &x > \frac{\sqrt{1-r^2}}{r}. \end{aligned} \tag{2.181}$$

Thus the normalized integral (equation (2.167)) becomes

$$\frac{Average\left\langle I_{tforced}\right\rangle}{Average\left\langle I_{tfree}\right\rangle} = \frac{1}{2}\int_{\frac{\sqrt{r^2-1}}{r}}^{1} \frac{(rx+\sqrt{(1-r^2)+r^2x^2})^2}{4\sqrt{(1-r^2)+r^2x^2}}dx. \tag{2.182}$$

This integral can again be split into three integrals

$$= \frac{r^2}{2}\int_{\frac{\sqrt{r^2-1}}{r}}^{1} \frac{x^2}{\sqrt{(1-r^2)+r^2x^2}}dx + r\int_{\frac{\sqrt{r^2-1}}{r}}^{1} x\,dx + \frac{1}{2}\int_{\frac{\sqrt{r^2-1}}{r}}^{1}\sqrt{(1-r^2)+r^2x^2}\,dx. \tag{2.183}$$

Gradshteyn and Ryzhik (1980) integral number 2.264.3 is used to evaluate the following three integrals . The first integral is equal to

$$\frac{\mathrm{r}^2}{2}\left[\frac{x}{2r^2}\sqrt{(1-r^2)+r^2x^2} - \frac{(1-r^2)}{2r^3}\ln(2r\sqrt{(1-r^2)+r^2x^2}+2r^2x)\right]_{\frac{\sqrt{r^2-1}}{2}}^{1} \tag{2.184}$$

.

$$=\frac{1}{4}\left[1+\frac{(1-r^2)}{r}\ln\left(\frac{\sqrt{r^2-1}}{r+1}\right)\right]. \qquad (2.185)$$

Again, the second integral is easily evaluated.

$$\mathrm{r}\int_{\frac{\sqrt{\mathrm{r}^2-1}}{r}}^{1} xdx = r\left[\frac{x^2}{2}\right]_{\frac{\sqrt{r^2-1}}{r}}^{1} = r\left[\frac{1}{2}-\frac{(r^2-1)}{2r^2}\right]$$
$$=\frac{r}{2}-\frac{r(r^2-1)}{2r^2}=\frac{r^2-r^2+1}{2r}=\frac{1}{2r}. \qquad (2.186)$$

Using Gradshteyn and Ryzhik (1980) integral number 2.262.1, the third integral is equal to

$$\frac{1}{2}\left[\frac{1}{2}x\sqrt{(1-r^2)+r^2x^2}+\frac{1}{2}\frac{(1-r^2)}{r}\ln(2r\sqrt{(1-r^2)+r^2x^2}+2xr^2)\right]_{\frac{\sqrt{r^2-1}}{r}}^{1}$$
$$=\left[\frac{1}{4}+\frac{1}{4}\frac{(1-r^2)}{r}\ln\left(\frac{2r(1+r)}{2r\sqrt{r^2-1}}\right)\right]. \qquad (2.187)$$

Combining the three integrals gives the averaged normalized intensity as

$$\frac{1}{4}+\frac{1}{4}\frac{(1-r^2)}{r}\ln\left(\frac{\sqrt{r^2-1}}{r+1}\right)+\frac{1}{2r}+\frac{1}{4}+\frac{1}{4}\frac{(1-r^2)}{r}\ln\left(\frac{(1+r)}{\sqrt{r^2-1}}\right)$$
$$=\frac{2}{4}+\frac{1}{2r}=\frac{1}{2}+\frac{1}{2r}=\frac{r+1}{2r}=\frac{1}{2}\left(1+\frac{1}{r}\right). \qquad (2.188)$$

2.14 Diffuse incidence when the media are different. In terms of Z,k.

In this section the case when the two media have different values of Z and k, is considered. The integration that is performed numerically to find the transmitted intensity over all angles is derived. From equation (2.133)

$$
\begin{aligned}
I_t &= \frac{\left|p_f\right|^2 Z_2 \left|k_{fx}+k_{rx}\right|^2}{\left|k_{rx}Z_2+\dfrac{k_1}{k_2}k_{tx}Z_1\right|^2 k_2}\mathrm{Re}(k_{tx}) \\
I_t &= \frac{\left|p_f\right|^2 \left|k_{fx}+k_{rx}\right|^2}{Z_2\left|k_{rx}+\dfrac{k_1}{k_2}k_{tx}\dfrac{Z_1}{Z_2}\right|^2 k_2}\mathrm{Re}(k_{tx}).
\end{aligned}
\tag{2.189}
$$

Thus

$$
\begin{aligned}
\frac{Z_2 I_t}{\left|p_f\right|^2} &= \frac{\left|k_{fx}+k_{rx}\right|^2}{\left|k_{rx}+\dfrac{k_1}{k_2}k_{tx}\dfrac{Z_1}{Z_2}\right|^2 k_2}\mathrm{Re}(k_{tx}) \\
&= \frac{\left|k_{fx}+k_{rx}\right|^2 \mathrm{Re}(\dfrac{k_{tx}}{k_2})}{\left|k_{rx}+\dfrac{k_1}{k_2}k_{tx}\dfrac{Z_1}{Z_2}\right|^2}.
\end{aligned}
\tag{2.190}
$$

From equations (2.110) to (2.112),

$$
k_{fx}=k_f\cos\theta_f=\sqrt{k_f^2-k_f^2\sin^2\theta_f}, \tag{2.191}
$$

$$
k_{rx}=k_r\cos\theta_r=\sqrt{k_1^2-k_f^2\sin^2\theta_f}, \tag{2.192}
$$

and

$$
k_{tx}=k_t\cos\theta_t=\sqrt{k_2^2-k_f^2\sin^2\theta_f}. \tag{2.193}
$$

Thus

$$
\begin{aligned}
\frac{Z_2 I_t}{\left|p_f\right|^2} &= \frac{\left|\sqrt{k_f^2-k_f^2\sin^2\theta_f}+\sqrt{k_1^2-k_f^2\sin^2\theta_f}\right|^2 \mathrm{Re}\left(\dfrac{\sqrt{k_2^2-k_f^2\sin^2\theta_f}}{k_2}\right)}{\left|\sqrt{k_1^2-k_f^2\sin^2\theta_f}+\dfrac{k_1}{k_2}\sqrt{k_2^2-k_f^2\sin^2\theta_f}\dfrac{Z_1}{Z_2}\right|^2} \\
&= \frac{\left|k_f\cos\theta_f+\sqrt{k_1^2-k_f^2\sin^2\theta_f}\right|^2 \mathrm{Re}\left(\sqrt{1-\dfrac{k_f^2}{k_2^2}\sin^2\theta_f}\right)}{\left|\sqrt{k_1^2-k_f^2\sin^2\theta_f}+\dfrac{k_1}{k_2}\sqrt{k_2^2-k_f^2\sin^2\theta_f}\dfrac{Z_1}{Z_2}\right|^2}.
\end{aligned}
\tag{2.194}
$$

Hence,

$$\frac{Z_2 I_t}{|p_f|^2} = \frac{\left|\frac{k_f}{k_1}\cos\theta_f + \sqrt{1-\frac{k_f^2}{k_1^2}\sin^2\theta_f}\right|^2 \mathrm{Re}(\sqrt{1-\frac{k_f^2}{k_2^2}\sin^2\theta_f})}{\left|\sqrt{1-\frac{k_f^2}{k_1^2}\sin^2\theta_f} + \frac{1}{k_2}\sqrt{k_2^2 - k_f^2\sin^2\theta_f}\,\frac{Z_1}{Z_2}\right|^2}. \tag{2.195}$$

Now

$$\frac{1}{k_2}\sqrt{k_2^2 - k_f^2\sin^2\theta_f} = \sqrt{1-\frac{k_f^2}{k_1^2}\frac{k_1^2}{k_2^2}\sin^2\theta_f}, \tag{2.196}$$

$$\mathrm{Re}\left(\sqrt{1-\frac{k_f^2}{k_1^2}\frac{k_1^2}{k_2^2}\sin^2\theta_f}\right) = \mathrm{Re}\left(\sqrt{1-\frac{k_f^2}{k_2^2}\sin^2\theta_f}\right), \tag{2.197}$$

and

$$\frac{k_f}{k_1} = r. \tag{2.198}$$

Thus

$$\frac{Z_2 I_t}{|p_f|^2} = \frac{\left|\frac{k_f}{k_1}\cos\theta_f + \sqrt{1-\frac{k_f^2}{k_1^2}\sin^2\theta_f}\right|^2 \mathrm{Re}\left(\sqrt{1-\frac{k_f^2}{k_1^2}\frac{k_1^2}{k_2^2}\sin^2\theta_f}\right)}{\left|\sqrt{1-\frac{k_f^2}{k_1^2}\sin^2\theta_f} + \frac{Z_1}{Z_2}\sqrt{1-\frac{k_f^2}{k_1^2}\frac{k_1^2}{k_2^2}\sin^2\theta_f}\right|^2}. \tag{2.199}$$

$$\text{Using} \qquad r = \frac{k_f}{k_1} \qquad \text{and defining} \qquad \alpha = \frac{k_1}{k_2} \qquad \beta = \frac{Z_1}{Z_2} \qquad \text{gives} \tag{2.200}$$

$$\frac{Z_2 I_t}{|p_f|^2} = \frac{\left|r\cos\theta_f + \sqrt{1-r^2\sin^2\theta_f}\right|^2 \mathrm{Re}\left(\sqrt{1-r^2\alpha^2\sin^2\theta_f}\right)}{\left|\sqrt{1-r^2\sin^2\theta_f} + \beta\sqrt{1-r^2\alpha^2\sin^2\theta_f}\right|^2}, \tag{2.201}$$

Using

$$\sin^2\theta_f = 1-\cos^2\theta_f, \tag{2.202}$$

gives

$$\begin{aligned}\frac{Z_2 I_t}{\left|p_f\right|^2} &= \frac{\left|r\cos\theta_f + \sqrt{1-r^2\left(1-\cos^2\theta_f\right)}\right|^2 \operatorname{Re}\left(\sqrt{1-r^2\alpha^2\left(1-\cos^2\theta_f\right)}\right)}{\left|\sqrt{1-r^2\left(1-\cos^2\theta_f\right)}+\beta\sqrt{1-r^2\alpha^2\left(1-\cos^2\theta_f\right)}\right|^2} \\ &= \frac{\left|r\cos\theta_f + \sqrt{1-r^2+r^2\cos^2\theta_f}\right|^2 \operatorname{Re}\left(\sqrt{1-r^2\alpha^2+r^2\alpha^2\cos^2\theta_f}\right)}{\left|\sqrt{1-r^2+r^2\cos^2\theta_f}+\beta\sqrt{1-r^2\alpha^2+r^2\alpha^2\cos^2\theta_f}\right|^2}.\end{aligned} \tag{2.203}$$

Integrating over all angles of incidence gives

$$\begin{aligned}&\int_0^{\frac{\pi}{2}} \frac{Z_2 I_t}{\left|p_f\right|^2}\sin\theta_f d\theta_f \\ &= \int_0^{\frac{\pi}{2}} \frac{\left|r\cos\theta_f + \sqrt{1-r^2+r^2\cos^2\theta_f}\right|^2 \operatorname{Re}\left(\sqrt{1-r^2\alpha^2+r^2\alpha^2\cos^2\theta_f}\right)}{\left|\sqrt{1-r^2+r^2\cos^2\theta_f}+\beta\sqrt{1-r^2\alpha^2+r^2\alpha^2\cos^2\theta_f}\right|^2}\sin\theta_f d\theta_f.\end{aligned} \tag{2.204}$$

Using

$$x = \cos\theta_f \text{ and } \qquad dx = -\sin\theta_f d\theta_f, \tag{2.205}$$

and noting that

$$\cos(0) = 1 \qquad \text{and} \qquad \cos(\frac{\pi}{2}) = 0, \tag{2.206}$$

gives, after reversing the limits in order to remove the minus sign from $dx = -\sin\theta_f d\theta_f$ the integral as

$$\int_0^1 \frac{\left|rx + \sqrt{1-r^2+r^2x^2}\right|^2 \operatorname{Re}\left(\sqrt{1-r^2\alpha^2+r^2\alpha^2x^2}\right)}{\left|\sqrt{1-r^2+r^2x^2}+\beta\sqrt{1-r^2\alpha^2+r^2\alpha^2x^2}\right|^2}dx. \tag{2.207}$$

This is the actual integration that is performed numerically to find the relative transmitted intensity over all angles.

2.15 Summary

As the equations in this chapter are quite complicated, initially the case of normal incidence is considered. Even in the case where the two media either side of the junction are the same, there will be both a transmitted and a reflected wave. This is because in the forced wave case only the medium in which the wave is incident will be driven by the forcing function. First the one dimensional wave equation is derived and solved for the case where the wave propagation in the incident medium is forced. This includes the inertia equation which is needed for deriving the relationship between the pressure and the particle velocity.

Next, the case of a normally incident forced wave (when the media are the same) is considered. The transmitted and reflected sound pressures are calculated. The incident intensity is then calculated considering both the incident wave and the reflected wave. The transmitted intensity is also calculated, and for future use the intensity of the reflected wave if it was traveling alone is also calculated. Then the normally incident case when the two media are different is considered. The same calculations as previously performed are repeated but this time with different values for the media properties on either side of the junction.

The oblique incidence case is then considered. It is shown that the transmitted nearfield that is produced in the case of total internal reflection transmits no power. The acoustical particle velocity for the oblique incident case is calculated followed by the calculations of transmitted and reflected pressures. Then the incident and reflected intensities are calculated. The incident intensity includes both the incident and the reflected wave.

The oblique incident values are used to calculate the diffuse field incidence case by averaging over all possible angles of incidence. When the media are the same this is done by analytical integration. Separate integrals need to be evaluated for the case when the forced wave number is less than or equal to the freely propagating wave number and for the case when the forced wave number is greater than or equal to the freely propagating wave number. For the diffuse field incidence case when the two media are different the equations which have to be evaluated numerically are derived.

Chapter 3 The fluid media sound results

3.1 Introduction

In this chapter, the theoretical results obtained in the previous chapter are presented as graphs so that the consequences can be easily interpreted. In particular, it is shown that the sum of the intensities propagated in the direction normal to the junction by the forced incident and freely propagating reflected waves are not equal to the transmitted intensity. This is because the cross terms in the intensity calculations do not vanish unless the incident wave is freely propagating. It is shown that the intensity incident on the junction is equal to the transmitted intensity if the cross terms are included. Where it is possible, the results are graphed in decibels. This is not always possible because some of the results are zero or change sign. The results for the simpler case of normal incidence are presented first, followed by the oblique incidence case. The results for the when the media on each side of the junction are the same, are presented first, followed by the case when the media have different impedances. Finally for oblique incidence the case when the impedances and wave numbers (wave speeds) are different is presented.

It should be noted that although the media are the same on each side of the junction, a discontinuity still exists because the forcing function which generates the forced incidence wave operates only in the first medium and not in the second medium, on the other side of the junction.

3.2 Normal incidence, the same media

The transmitted intensity is

$$I_t(r) = \frac{\left|p_f\right|^2}{Z}\left(\frac{k+k_f}{2k}\right)^2 = I_{(i+r)}(r), \tag{3.1}$$

from equations (2.38) and (2.45), where

$$Z = \rho_0 c. \tag{3.2}$$

Put

$$r = \frac{k_f}{k}. \tag{3.3}$$

For a forced incident wave the transmitted intensity is

$$I_t(r) = \frac{|p_f|^2}{Z}\left(\frac{1+r}{2}\right)^2 = I_{(i+r)}(r). \tag{3.4}$$

for a freely propagating wave (r=1), the transmitted intensity is

$$I_t = \frac{|p_f|^2}{Z} = I_{(i+r)}(1) = I_i(1). \tag{3.5}$$

The last equality occurs because the media are the same and hence there is no reflected wave when the incident wave is freely propagating. The normalized transmitted intensity is

$$\frac{I_t(r)}{I_t(1)} = \left(\frac{1+r}{2}\right)^2 = \frac{I_{(i+r)}(r)}{I_{(i+r)}(1)}. \tag{3.6}$$

Equation (3.6) is shown in Figure 3.1 in decibels. The freely propagating case is given by r=1. For the r is less than one case, the transmitted intensity is less than the intensity for a freely propagating incident wave (r=1).This figure shows that attenuation of a normally incident forced wave is at most 6 dB. This is more than the attenuation of a normally incident freely propagating wave. It should also be noted that the graph is independent of frequency. In the r is greater than one case the intensity of the transmitted wave is greater than that for the freely propagating case when r=1.

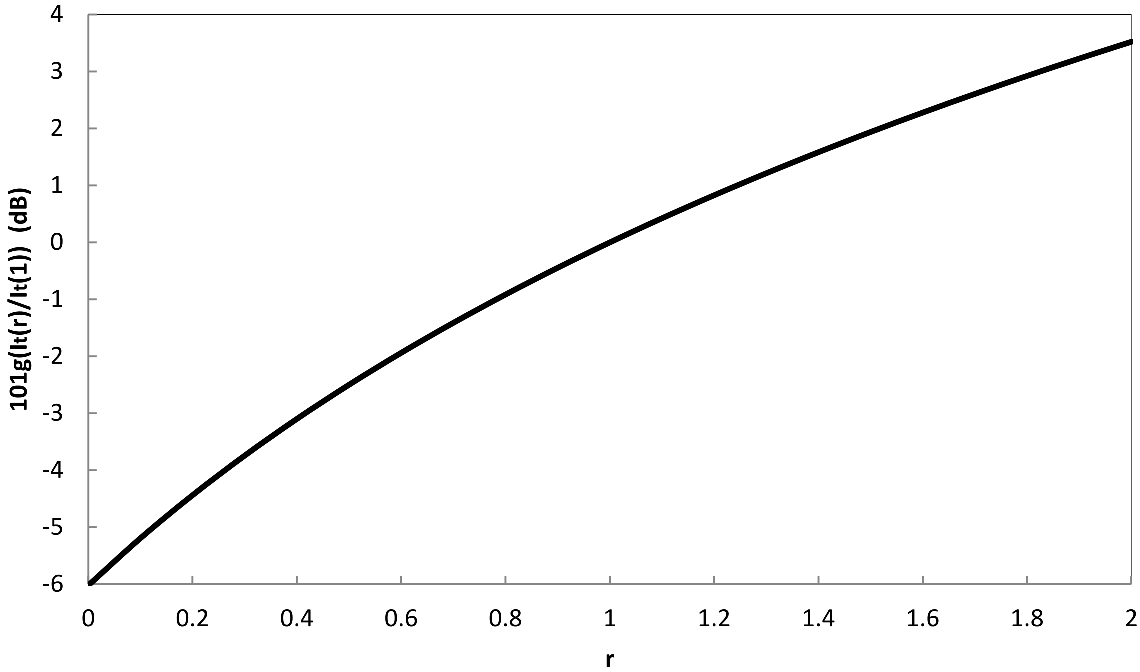

Figure 3.1: The transmitted intensity due to a normally incident forced wave.

The ratio of the transmitted (or total) intensity to the transmitted intensity when the incidence wave is freely propagating goes to infinity when the wave number ratio (r) goes to infinity because power that the forcing function has to inject into the first medium in order to generate the forced incident wave of constant amplitude goes to infinity as the wave number ratio goes to infinity. In other words, it is easiest to generate a wave with the freely propagating wave number and becomes harder to generate the wave when its wave number is very different from the freely propagating wave number.

The incident intensity in the absence of the reflected wave is,

$$I_i(r) = \frac{k_f |p_f|^2}{\rho_0 \omega} = \frac{k_f}{k}\frac{|p_f|^2}{Z} = r\frac{|p_f|^2}{Z}. \tag{3.7}$$

The reflected intensity in the positive x direction in the absence of the incidence wave is, from equation (2.52)

$$I_r(r) = -\frac{|p_f|^2}{Z}\left(\frac{1-r}{2}\right)^2. \tag{3.8}$$

For the freely propagating wave (r=1)

$$I_i(1) = \frac{|p_f|^2}{Z}. \tag{3.9}$$

The intensities are now normalized by dividing them by the intensity of the freely propagating incident wave

$$\frac{I_i(r)}{I_i(1)} = r. \tag{3.10}$$

From equations (3.8) and (3.9) the normalized reflected intensity is

$$\frac{I_r(r)}{I_i(1)} = -\left(\frac{1-r}{2}\right)^2 = -\left(\frac{r-1}{2}\right)^2. \tag{3.11}$$

From equations (3.4) and (3.9) the normalized transmitted intensity is

$$\frac{I_t(r)}{I_i(1)} = \left(\frac{1+r}{2}\right)^2. \tag{3.12}$$

Thus the sum of the incident and reflected intensities (where positive denotes energy being propagated in the positive x-axis direction) is given by

$$\begin{aligned} \frac{I_i(r)+I_r(r)}{I_i(1)} &= r - \left(\frac{r-1}{2}\right)^2 \\ &= \frac{4r - r^2 + 2r - 1}{4} \\ &= \frac{-r^2 + 6r - 1}{4}. \end{aligned} \tag{3.13}$$

Note that equation (3.13) is not equal to the transmitted intensity

$$\frac{I_t(r)}{I_i(1)} = \frac{r^2 + 2r + 1}{4}, \tag{3.14}$$

unless the incident wave is freely propagating (r=1). These normalized intensities are shown in Figure 3.2.

Thus the power propagated by the incident and reflected waves cannot be calculated separately unless the incident wave is freely propagating. The actual incident power on the incidence side of the junction plane at x equals 0 can only be calculated as $I_{(i+r)}(r)$ and cannot be calculated as the sum of $I_i(r)$ and $I_r(r)$ unless the incident wave is freely propagating.

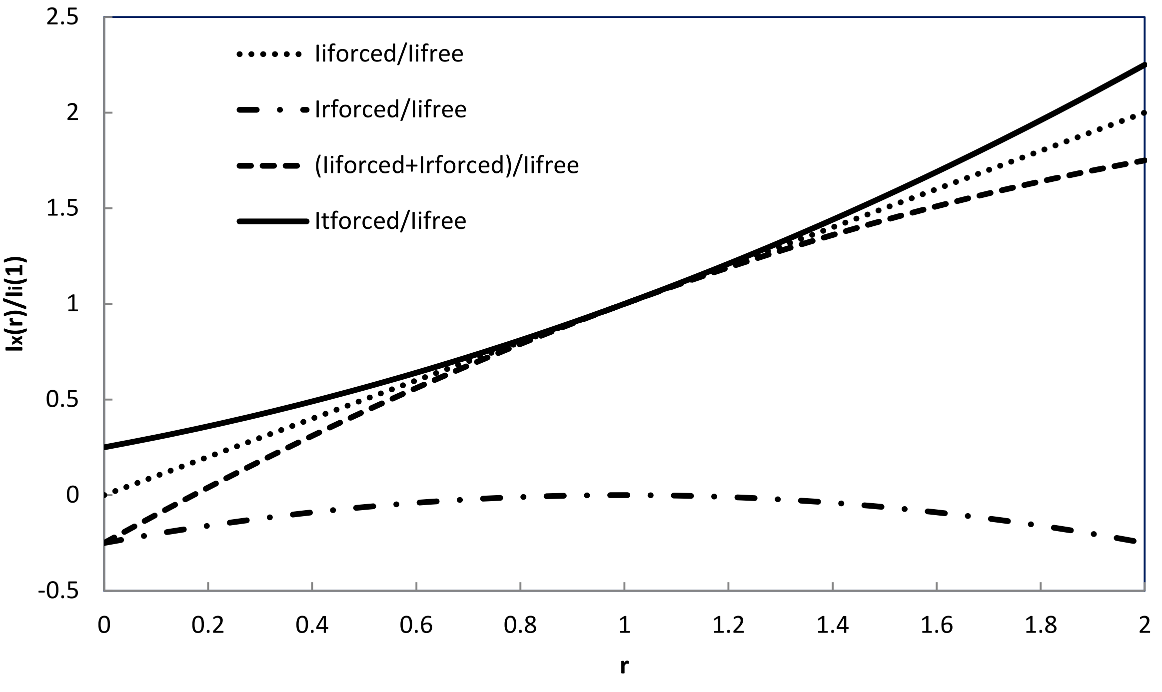

Figure 3.2: The transmitted, reflected, incident and sum of incident and reflected intensities due to a normally incident forced wave.

In Figure 3.2 $I_{iforced}/I_{ifree}$ is the normalised incident wave intensity calculated on its own, in the absence of the reflected wave. $I_{rforced}/I_{ifree}$ is the normalised reflected wave intensity propagated in the positive x direction. Hence it is always negative or zero. If these two are added together to obtain $(I_{iforced}+I_{rforced})/I_{ifree}$, it is noted that this sum does not equal $I_{tforced}/I_{ifree}$. This shows that the intensity in the first medium which must be equal to the intensity in the second medium, cannot be calculated correctly by considering the forced incident and reflected wave separately.

The ratios of the transmitted and reflected pressure to the forced incident pressure are as follows.

From equation (2.27)

$$\frac{p_t}{p_f} = \frac{r+1}{2}. \tag{3.15}$$

From equation (2.28)

$$\frac{p_r}{p_f} = \frac{r-1}{2}. \tag{3.16}$$

The sum of these two equations is

$$\frac{p_f + p_r}{p_f} = 1 + \frac{r-1}{2} = \frac{r+1}{2} = \frac{p_t}{p_f}. \tag{3.17}$$

The transmitted, reflected and incidence pressures are graphed in Figure 3.3.

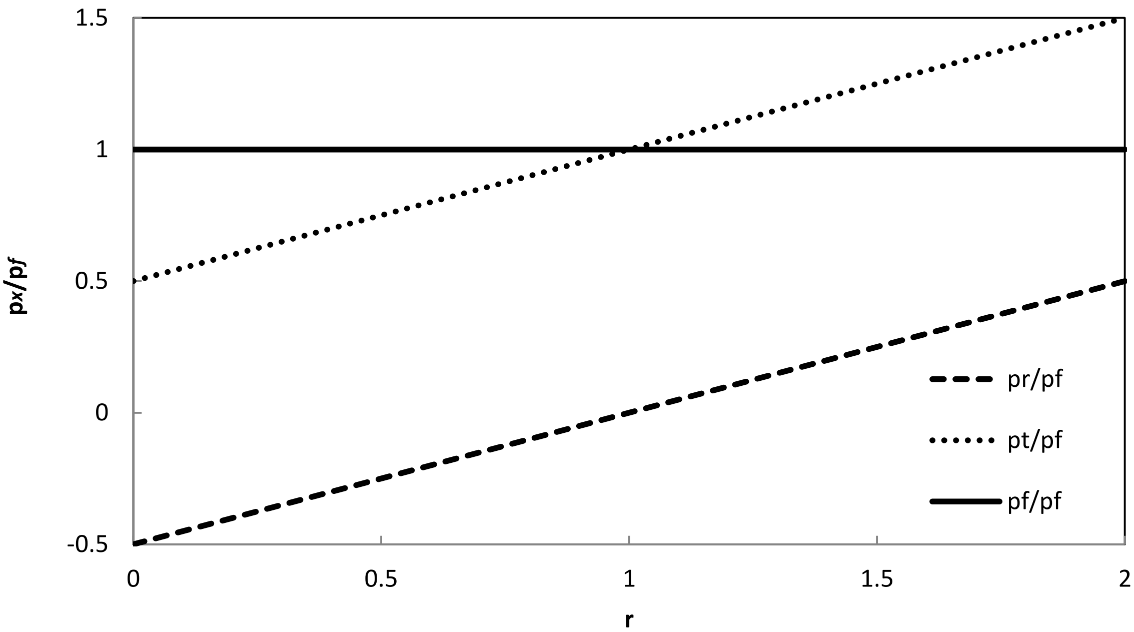

Figure 3.3: The normalised transmitted, reflected and incident sound pressures as a function of the ratio *r* of the forced incident wave number to the freely propagating wave number.

Figure 3.3 shows that the transmitted pressure is less than or greater than the normally incident forced pressure.

3.3 Normal incidence, different media.

From equation (2.156) for a normally incident wave when the two media are different equations (2.67) and (2.72) gives

$$I_t(r) = I_{(i+r)}(r) = (r+1)^2 \frac{z_1}{(z_2 + z_1)^2} \left| p_f \right|^2. \tag{3.18}$$

Thus

$$I_t(1) = I_{(i+r)}(1) = \frac{4z_2}{(z_2 + z_1)^2} \left| p_f \right|^2, \tag{3.19}$$

and

$$\frac{I_t(r)}{I_t(1)} = \frac{I_{(i+r)}(r)}{I_{(i+r)}(1)} = \left(\frac{r+1}{2} \right)^2. \tag{3.20}$$

Equation (3.20) is the same as equation (3.6). Thus figure 3.1 also applies for normal incidence when the media are different.

3.4 Oblique incidence, same media.

From equation (2.156), the average transmitted intensity for the case of a freely propagating diffuse incident field is

$$\left\langle I_t(1) \right\rangle = \frac{\left| p_f \right|^2}{2Z}, \tag{3.21}$$

where p_f is the rms sound pressure of the incident diffuse sound field. The value of equation (3.21) will be used to normalize the other values.

From equation (2.150) and (3.21)

$$\frac{I_t(r,\theta)}{\langle I_t(1)\rangle}=\begin{cases}\dfrac{(r\cos\theta+\sqrt{1-r^2\sin^2\theta})^2}{2\sqrt{1-r^2\sin^2\theta}} & if \quad r|\sin\theta|<1 \\ 0 & if \quad r|\sin\theta|\geq 1\end{cases}. \tag{3.22}$$

From equation (2.180) and (2.188), the normalized transmitted intensity due to diffusely incident forced sound waves when the media are the same is given by

$$\frac{<I_t(r)>}{<I_t(1)>}=\begin{cases}(1+r)/2 & \text{if r} \leq 1 \\ (1+1/r)/2 & \text{if r} \geq 1.\end{cases} \tag{3.23}$$

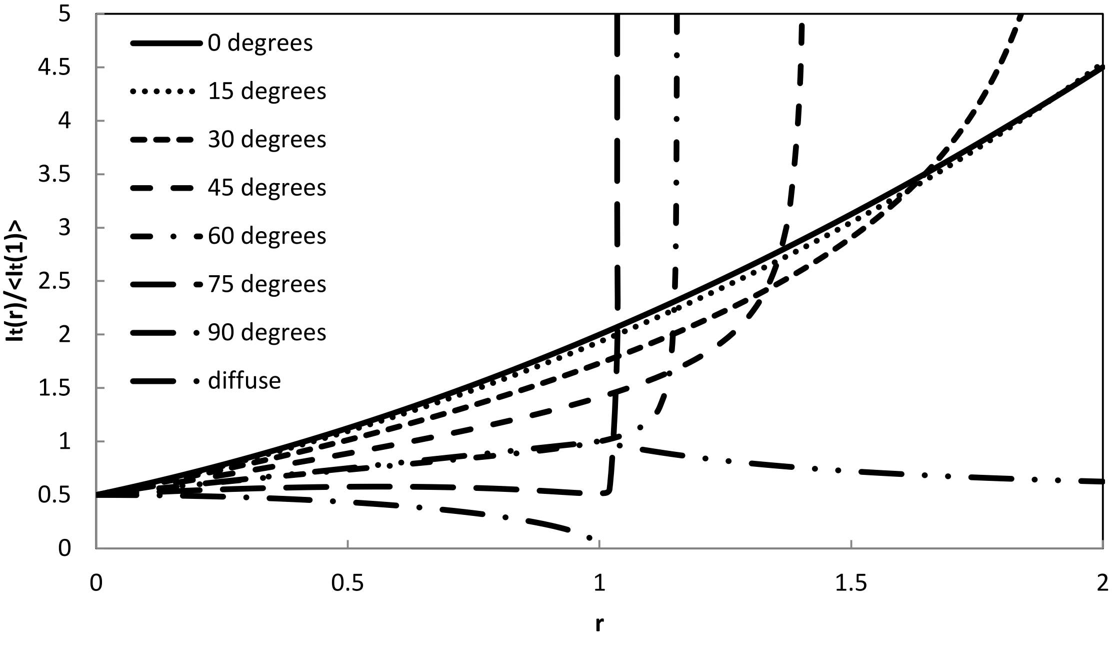

Figure 3.4 The normalised transmitted intensity due to forced plane waves incident at angles of incidence from 0 to 90 degrees in 15 degree increments.

Equations (3.22) and (3.23) are graphed in figure 3.4. For values of r greater than one, total internal reflection is seen to occur for the larger angles. The zero values of intensity which occur because of total internal reflection are not graphed in figure 3.4. It is interesting to note that the curves tend to infinity as the value of r for which total internal reflection occurs is approached from the direction of r equals zero, except for the 90 degree incident case. The diffuse field curve is equal to 0.5 at r=0 and tends to 0.5 as r tends to infinity. Thus for the diffuse field incidence case, the forced transmitted intensity is less than or equal to the freely propagating transmitted intensity by at most 3 dB. It should be noted that all the curves are equal when r equals zero. This is because the angle of incidence cannot really be defined when the incident forced wave number is equal to zero. It should be noted that the normalization used in figure 3.4 is different from that used in figure 3.1.

3.5 Oblique incidence, different impedances, same wave numbers

From equation (2.203), the normalized transmitted intensity, for an obliquely incident plane wave when the media are different, is given by

$$\frac{Z_2 I_t(r)}{\left|p_f\right|^2} = \frac{\left|r\cos\theta + \sqrt{1-r^2+r^2\cos^2\theta}\right|^2 \operatorname{Re}\left(\sqrt{1-r^2\alpha^2+r^2\alpha^2\cos^2\theta}\right)}{\left|\sqrt{1-r^2+r^2\cos^2\theta}+\sqrt{1-r^2\alpha^2+r^2\alpha^2\cos^2\theta}\,\beta\right|^2}, \tag{3.24}$$

where $r = \dfrac{k_f}{k_1}$, $\alpha = \dfrac{k_1}{k_2}$ $\beta = \dfrac{Z_1}{Z_2}$.

If the wave numbers of freely propagating waves in the two media are the same (α=1), then equation (3.24) becomes

$$\frac{Z_2 I_t(r)}{\left|p_f\right|^2} = \frac{\left|r\cos\theta + \sqrt{1-r^2\sin^2\theta}\right|^2 \operatorname{Re}\left(\sqrt{1-r^2\sin^2\theta}\right)}{\left|1+\beta\right|^2 \left|\sqrt{1-r^2\sin^2\theta}\right|^2}. \tag{3.25}$$

Thus

$$\frac{\left|1+\beta\right|^2 Z_2 I_t(r)}{2\left|p_f\right|^2} = \begin{cases} \dfrac{\left(r\cos\theta + \sqrt{1-r^2\sin^2\theta}\right)^2}{2\sqrt{1-r^2\sin^2\theta}} & \text{if } r\left|\sin\theta\right| \le 1 \\ 0 & \text{if } r\left|\sin\theta\right| \ge 1 \end{cases}. \tag{3.26}$$

The right hand side of equation (3.26) is the same as the right side of equation (3.22). Hence averaging over a diffuse field incidence gives

$$\frac{\left|1+\beta\right|^2 Z_2 \left\langle I_t(r)\right\rangle}{2\left|p_f\right|^2} = \begin{cases} (1+r)/2 & \text{if } r \le 1 \\ (1+1/r)/2 & \text{if } r \ge 1. \end{cases} \tag{3.27}$$

Thus

$$\frac{\left|1+\beta\right|^2 Z_2 \left\langle I_t(r)\right\rangle}{2\left|p_f\right|^2} = 1. \tag{3.28}$$

Dividing equation (3.26) by (3.21) gives

$$\frac{I_t(r)}{\left\langle I_t(1)\right\rangle} = \begin{cases} \dfrac{\left(r\cos\theta + \sqrt{1-r^2\sin^2\theta}\right)^2}{2\sqrt{1-r^2\sin^2\theta}} & \text{if } r\left|\sin\theta\right| \le 1 \\ 0 & \text{if } r\left|\sin\theta\right| \ge 1 \end{cases}. \tag{3.29}$$

Dividing equation (3.27) by (3.28) gives

$$\frac{\left\langle I_t(r)\right\rangle}{\left\langle I_t(1)\right\rangle} = \begin{cases} (1+r)/2 & \text{if } r \le 1 \\ (1+1/r)/2 & \text{if } r \ge 1. \end{cases} \tag{3.30}$$

Equation (3.29) is the same as equation (3.22). Equation (3.30) is the same as equation (3.23). Hence figure 3.4 also applies when the wave numbers of the two media are the same even if the characteristic impedances are different.

3.6 Oblique incidence, different wave numbers

For the case of different freely propagating wave numbers in the two media, equation (3.24) applies. However this equation will be multiplied by two before graphing it to give

$$\frac{2Z_2 I_t(r)}{\left|p_f\right|^2} = \frac{2\left|r\cos\theta + \sqrt{1-r^2+r^2\cos^2\theta}\right|^2 \operatorname{Re}\left(\sqrt{1-r^2\alpha^2+r^2\alpha^2\cos^2\theta}\right)}{\left|\sqrt{1-r^2+r^2\cos^2\theta}+\beta\sqrt{1-r^2\alpha^2+r^2\alpha^2\cos^2\theta}\right|^2}. \tag{3.31}$$

This means that the transmitted intensity is normalized by dividing it by half the intensity of a plane sound wave with the same sound pressure as the incident plane sound wave but propagating in the second medium. This normalization is used because $<I_t(1)>$ cannot be calculated analytically. The freely propagating wave numbers in the two media are different, and thus the integration needs to be performed numerically in this case. The normalization is given by equation (3.21) , except that in this case it is necessary to choose which media's impedance to use because they are now different. The normalization is similar to that which can be deduced from equation (3.28), except that the factor $\left|1+\beta\right|^2$ is not used. The normalized transmitted intensity is plotted as a function of the ratio r of the forced incident wave number to the freely propagating wave number in the first medium for angles of incidence in 15 degree increments from 0 to 90 degrees. The ratio of the freely propagating wave number in the first medium to that in the second medium (α) is given the values of ½ or 2. The α equals 1 case is covered in the previous section. The ratio of the characteristic impedance of the first medium to that of the second medium (β) is given the values of ½, 1 and 2. The resulting graphs are shown in figure 3.5 through to figure 3.10.

From equation (2.207)

$$\frac{Z_2\langle I_t(r)\rangle}{\left|p_f\right|^2}=\int_0^1 \frac{\left|rx+\sqrt{1-r^2+r^2x^2}\right|^2 \mathrm{Re}(\sqrt{1-r^2\alpha^2+r^2\alpha^2x^2})}{\left|\sqrt{1-r^2+r^2x^2}+\sqrt{1-r^2\alpha^2+r^2\alpha^2x^2}\,\beta\right|^2}dx. \tag{3.32}$$

Equation (3.32) is graphed in figure 3.11 and figure 3.12 where it is normalized by dividing it by $<I_t(1)>$ which is the transmitted intensity due to a freely propagating incident diffuse field. This is the same normalization as used in equations (3.30) and (3.23). The only difference is that in this case the integrations need to be performed numerically.

The numerical integrations were performed in Matlab using the “quadgk” quadrature function to perform the integrations. When total internal reflection occurs the integral in equation (3.32) needs only to be evaluated from x equals $\sqrt{1-1/(r\alpha)^2}$ to x equals 1.

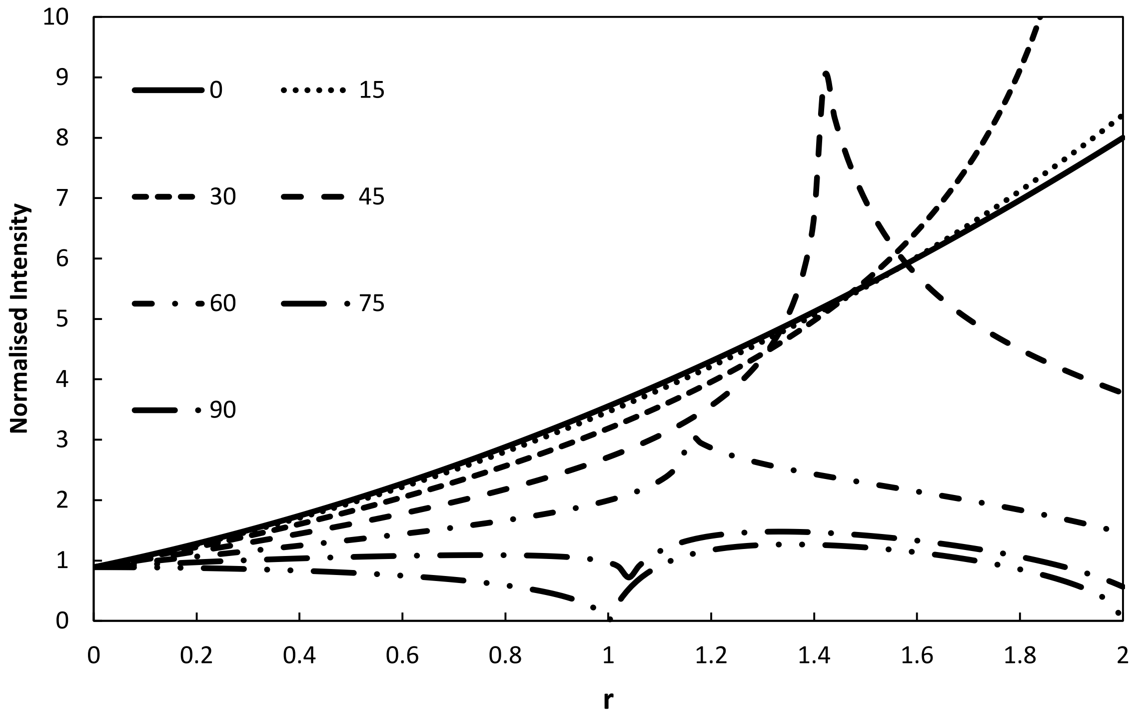

Figure 3.5 The transmitted intensity due to a plane sound wave incident at angles ranging from 0 to 90 degrees in 15 degree increments. The results are graphed as a function of the ratio *r* of the forced incident wave number to the wave number of a freely propagating wave in the first medium. The ratio of the wave number of a freely propagating wave in the first medium to that in the second medium (α) is ½. The ratio of the impedance of the characteristic impedance of the first medium to that of the second medium (β) is ½.

In figure 3.5 the local minima and maxima are shown in the range from *r* is equal to 0 to 2. There is a minimum of zero at 90 degree curve. The 45 and 60 degree curves have local maximums. The 75 and 90 degree curves have local minimums. The slopes of the 0, 15 and 30 degree curves increase from 0 to 2.

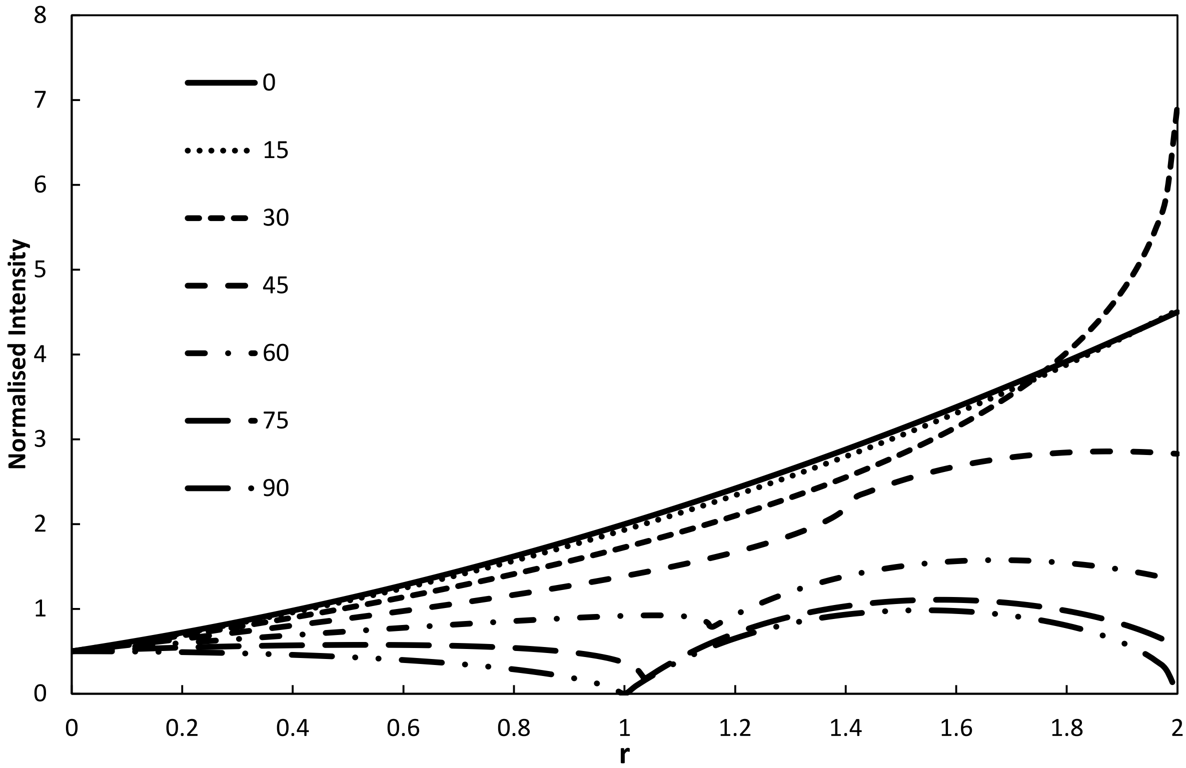

Figure 3.6 The transmitted intensity due to a plane sound wave incident at angles ranging from 0 to 90 degrees in 15 degree increments. The results are graphed as a function of the ratio *r* of the forced incident wave number to the wave number of a freely propagating wave in the first medium. The ratio of the wave number of a freely propagating wave in the first medium to that in the second medium (α) is ½. The ratio of the impedance of the characteristic impedance of the first medium to that of the second medium (β) is 1.

In figure 3.6, 0, 15, and 30 degree curves increase from 0 to 2, and the 60, 75, and 90 degree curves have a local minima.

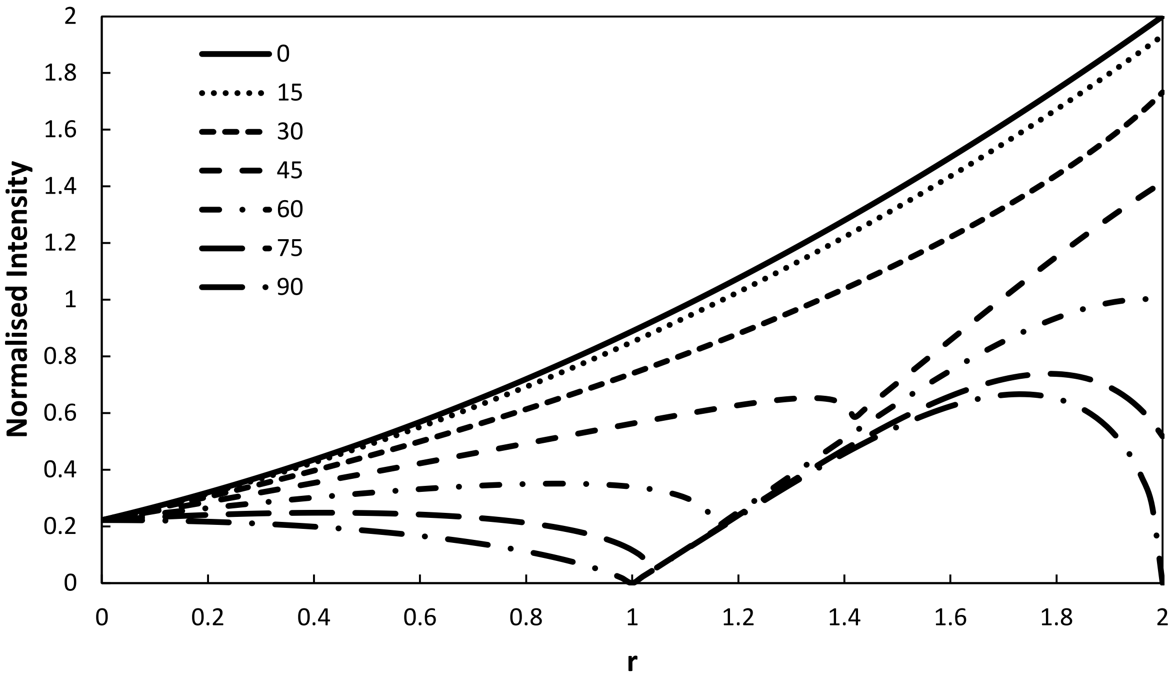

Figure 3.7 The transmitted intensity due to a plane sound wave incident at angles ranging from 0 to 90 degrees in 15 degree increments. The results are graphed as a function of the ratio r of the forced incident wave number to the wave number of a freely propagating wave in the first medium. The ratio of the wave number of a freely propagating wave in the first medium to that in the second medium (α) is ½. The ratio of the characteristic impedance of the first medium to that of the second medium (β) is 2.

In figure 3.7, the 0, 15 and 30 degree curves all have increasing slopes. The 45, 60, 75, and 90 degree curves all have a local minima. The 75 and 90 degree curves display a local maxima between r=1 and r=2. The 90 degree curves goes to zero at r=2.

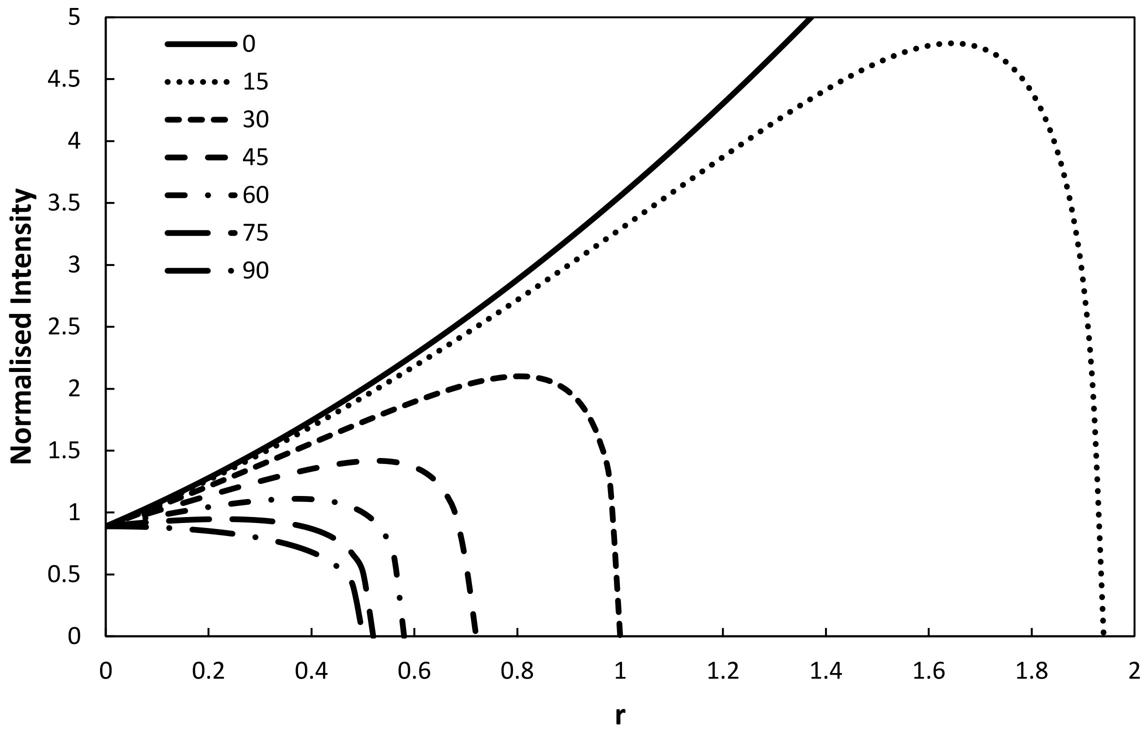

Figure 3.8 The transmitted intensity due to a plane sound wave incident at angles ranging from 0 to 90 degrees in 15 degree increments. The results are graphed as a function of the ratio *r* of the forced incident wave number to the wave number of a freely propagating wave in the first medium. The ratio of the wave number of a freely propagating wave in the first medium to that in the second medium (α) is 2. The ratio of the impedance of the characteristic impedance of the first medium to that of the second medium (β) is 1/2.

In figure 3.8, the 0 degree curve displays an increasing slope. The 15, 30, 45, 60 and 75 degree curves have a local maximum and then go to zero. The 90 degree curve slope decreases to zero. The 15 degree curve has its peak around $r=1.7$. The other curves all have lower values of *r* for their peak. The greater the angle, the lower the *r* value for which the peak occurs.

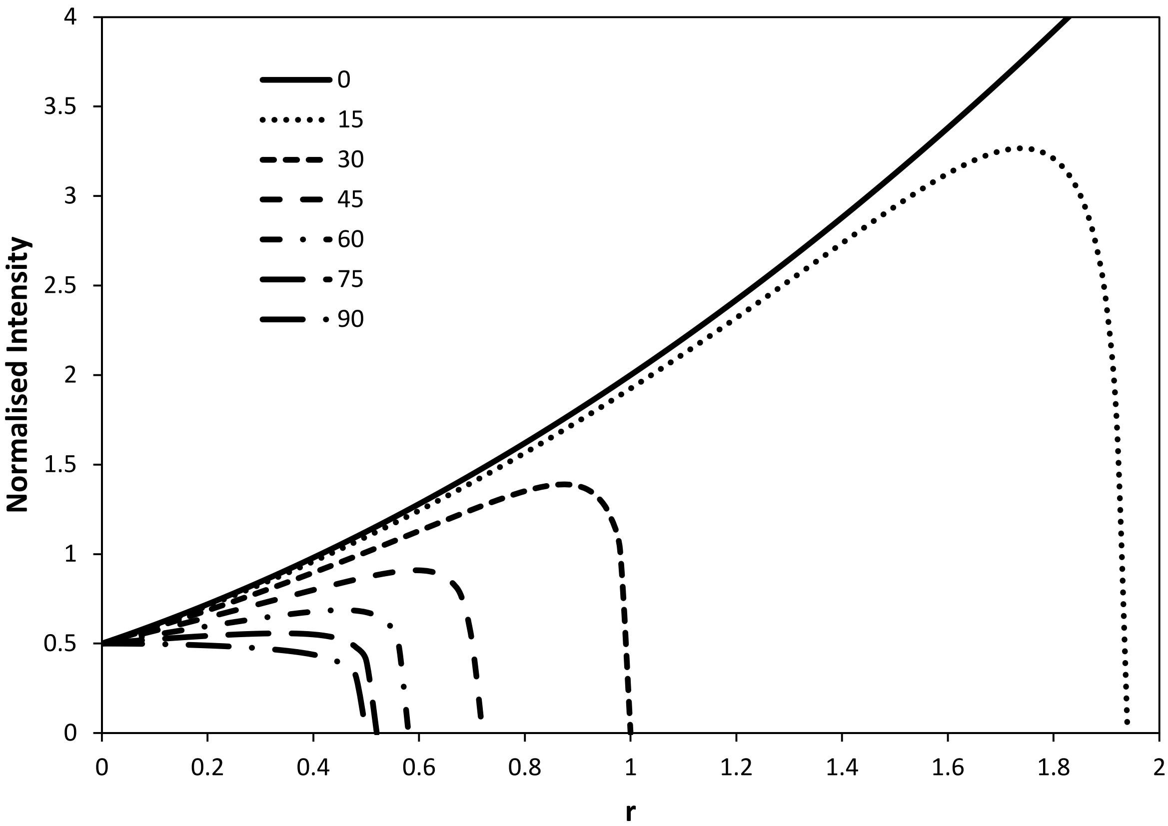

Figure 3.9 The transmitted intensity due to a plane sound wave incident at angles ranging from 0 to 90 degrees in 15 degree increments. The results are graphed as a function of the ratio *r* of the forced incident wave number to the wave number of a freely propagating wave in the first medium. The ratio of the wave number of a freely propagating wave in the first medium to that in the second medium (α) is 2. The ratio of the impedance of the characteristic impedance of the first medium to that of the second medium (β) is 1.

Figure 3.9 is very similar to figure 3.8 in its appearance. However the peaks in the 15, 30, 45, 60 and 75 degree curves have a smaller value of intensity. The 0 degree curve displays an increasing slope. The 15, 30, 45, 60 and 75 degree curves display a maximum and then they go to zero as *r* increases. The 90 degree curve decreases to zero. The 15 degree curve has its peak around r=1.78. The other curves are all have lower values of *r* for their peak. The greater the angle, the lower the *r* value for which the peak occurs.

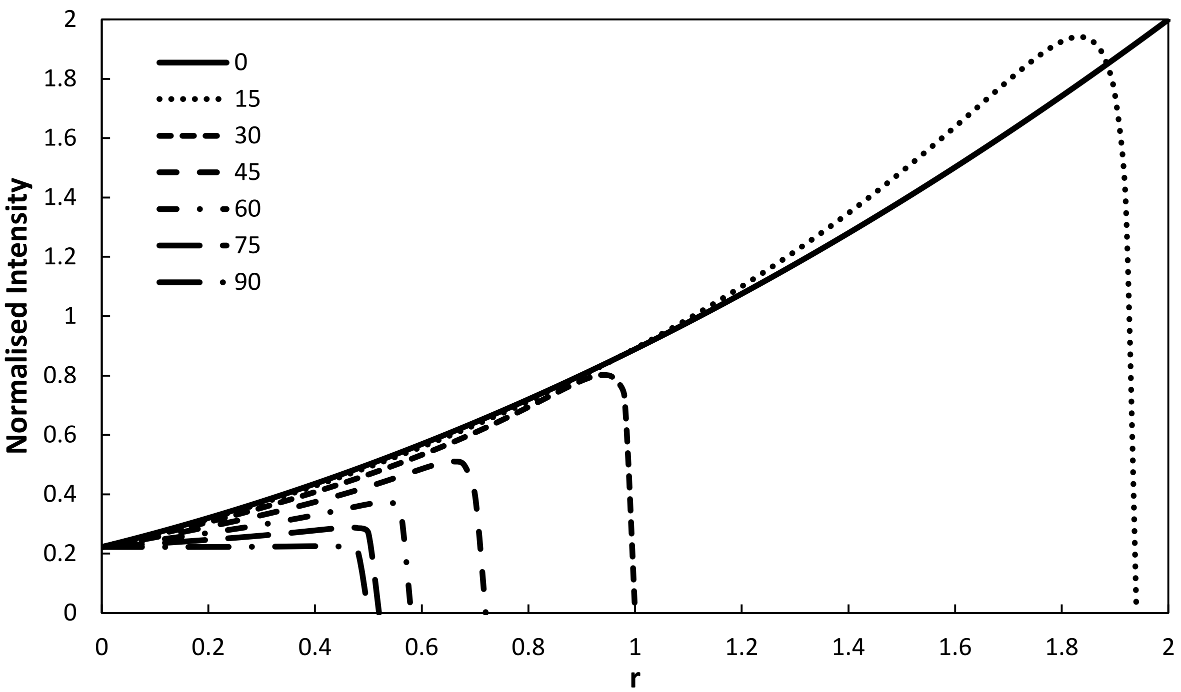

Figure 3.10 The transmitted intensity due to a plane sound wave incident at angles ranging from 0 to 90 degrees in 15 degree increments. The results are graphed as a function of the ratio r of the forced incident wave number to the wave number of a freely propagating wave in the first medium. The ratio of the wave number of a freely propagating wave in the first medium to that in the second medium (α) is 2. The ratio of the impedance of the characteristic impedance of the first medium to that of the second medium (β) is 2.

Figure 3.10 looks very much like figure 3.9. Again the difference is that the intensity is much lower. The 0 degree curve displays an increasing slope. The 15, 30, 45, 60 and 75 degree curves display a maximum and then go to zero. The 90 degree curve decreases to zero. The 15 degree curve has its peak around r=1.82. The other peaks all have lower values of r for their peaks. The greater the angle, the lower the r value for which the peak occurs.

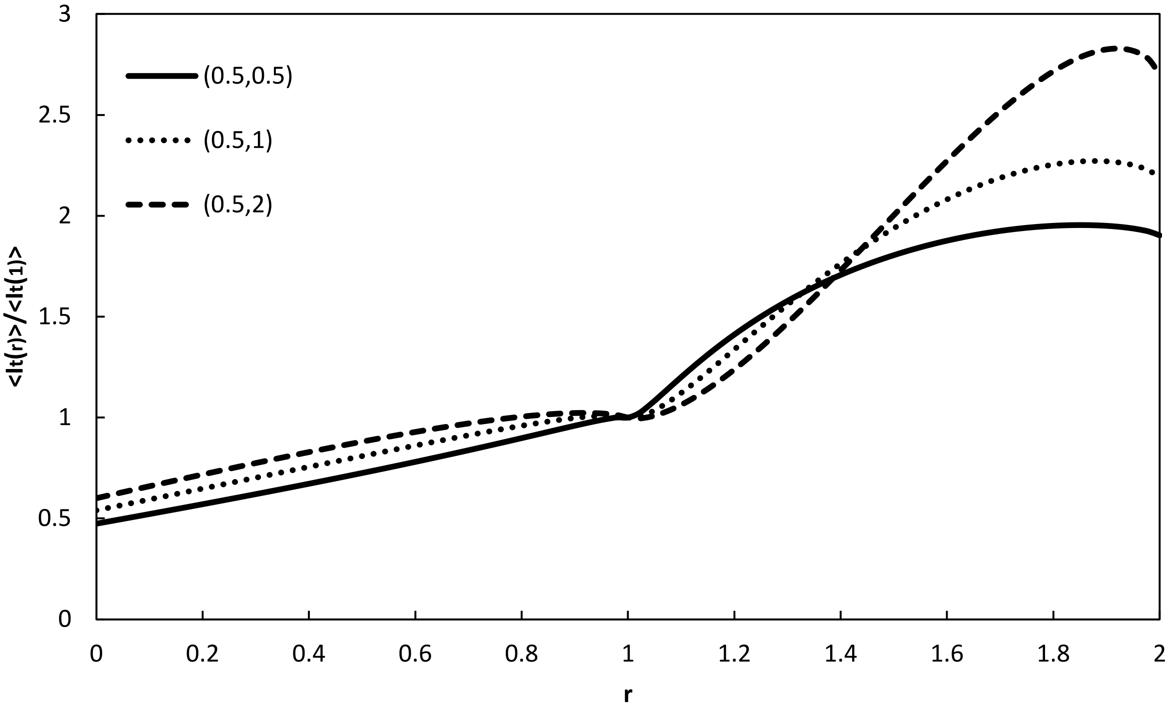

Figure 3.11 The transmitted intensity due to a forced diffuse incident sound field. The results are graphed as a function of the ratio *r* of the forced incident wave number to the wave number of a freely propagating wave in the first medium. Alpha is equal to ½. Beta is ½, 1 or 2.

For r=0 the values in figure 3.11 are close to 0.5. They increase as r increases with a slight minima at r=1. For r greater than 1, the values are greater than 1 and have a maxima just below r=2.

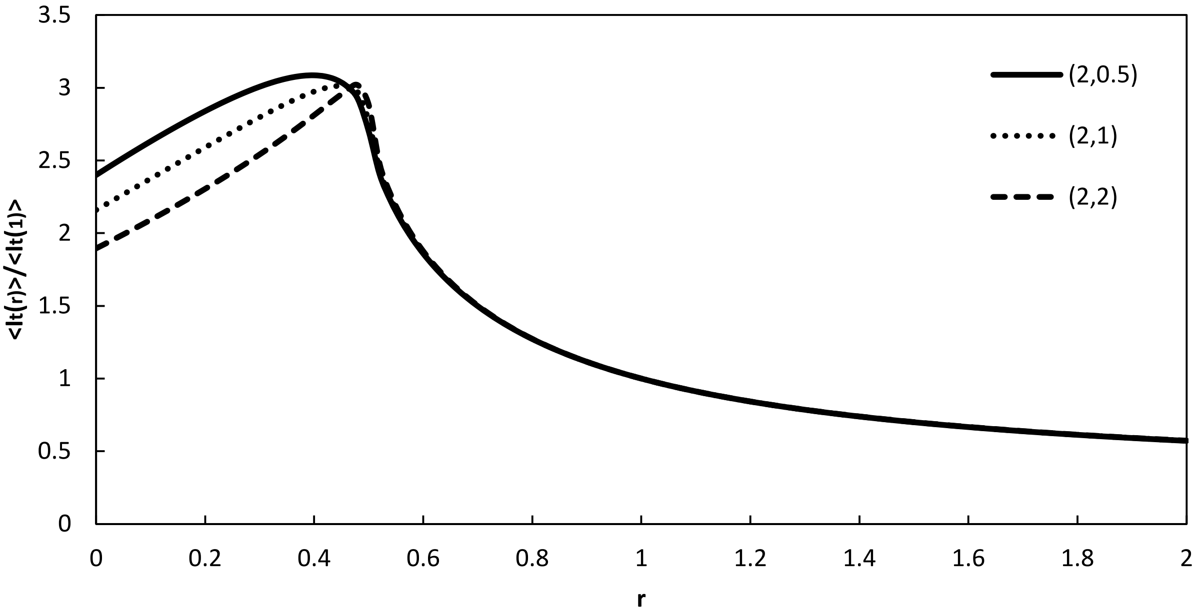

Figure 3.12 The transmitted intensity due to a forced diffuse incident sound field. The results are graphed as a function of the ratio *r* of the forced incident wave number to the wave number of a freely propagating wave in the first medium. Alpha is equal to 2. Beta is ½, 1, or 2.

In figure 3.12 all the curves have a maximum at about r=0.5. The curves follow different paths below r=0.5. The lines are almost the same for values of r greater than 0.5. The values are greater than 1 for r less than 1 and less than 1 for values of r greater than 1.

Looking back at the earlier graphs, namely figure 3.8 tofigure 3.10, the decreasing values above r=0.5 are due to total internal reflection or some of the angles of incidence. The behavior in figure 3.12 is qualitatively the opposite of what happens infigure 3.11. In figure 3.4, which is the alpha equals one case, the diffuse field values have a peak of 1 at r=1 and are less than one when r is less than or greater than 1.

3.7 Summary

Initially the case of a normally incident wave with the same medium on either side of the junction is considered. Note that transmission and reflection occur at the junction because the medium is driven by the forcing function on only one side of the junction. Figures 3.1-3.3 display a graph of intensity versus the forced wave number divided by the freely propagating wave number. Figure 3.2 shows the ratio of the separately calculated incident and reflected intensities, and the intensity carried by the combination of the incident and reflected waves. This shows that the incident intensity cannot be calculated by separately calculating the intensities of the forced and reflected waves. Figure 3.3 shows the values of the incident and reflected transmitted pressures. Then the case of a normally incident wave with different media on either side of the junction is considered. Next, the case of an oblique incidence wave is considered when the media on either side of the junction are the same. A graph of intensity versus r is produced showing the incident angles of 0, 15, 30, 45, 60, 75, and 90 degrees. The oblique incidence wave is again considered but with different impedances on either side of the junction. The same wave numbers are on either side of the junction. Again the oblique incidence wave case is considered but this time with different wave numbers on either side of the junction. Figures 3.5-3.12 are produced. They show the normalised intensity versus r. The graphs display lines showing the incident angles of 0, 15, 30, 45, 60, 75, and 90 degrees. The graphs show every possible combination of ($\alpha=(k_1/k_2)$) equal to 0.5, 1 or 2, and ($\beta=(Z_1/Z_2)$) equal to 0.5, 1 or 2.

Chapter 4 The transmission of bending waves between two panels at a pinned joint.

4.1 Introduction

In this chapter, the bending wave intensity transmitted at the pinned line junction of two infinite half plates, when a forced bending wave is incident on the line junction, is calculated. The normally incident case is considered first, followed by the oblique incident case. Then the case of a diffuse incident vibration field is considered. Finally, the case when the incident vibration field is excited by a diffuse acoustic field is studied.

4.2 The transmitted and reflected wave equations for normal incidence with a freely propagating incident wave.

Two half infinite flat plates lying in the y=0 plane, are rigidly connected at a pinned joint along the z-axis (see figure 4.1). A freely propagating transverse bending wave in plate one moving in the positive x direction is incident normally on the z axis. Four things need to be found. What are the ratios of the amplitudes of the propagating reflected wave (r), the nearfield reflected wave (r_j), the propagating transmitted wave (t) and the nearfield transmitted wave (t_j) to the amplitude of the freely propagating incident wave. These four unknowns will now be found using much substitution, differentiation and simultaneous equations.

Two half infinite plates joined rigidly at a junction are being considered. They are rigidly connected at a pin joint. The plates may rotate about the pin joint. However they are constrained by the pin to have zero transverse velocity at the joint (they cannot move up and down at the joint). The two plates are in line with each other.

Plate 1 is in the half infinite plane *y=0*, and *x* is less than or equal to zero. Plate 2 is in the half infinite plane *y=0* and *x* is greater than or equal to zero. The two plates are joined at *x=0* and *y=0*. Please see the diagram below.

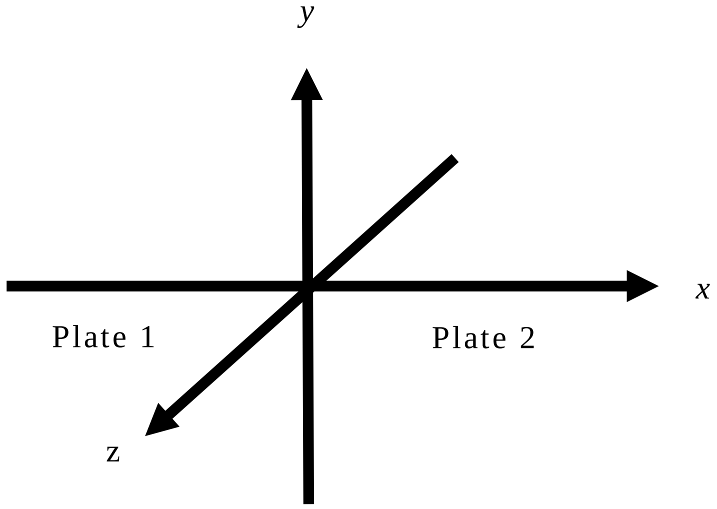

Figure 4.1 Plate 1 and Plate 2.

It is assumed that the oscillations of the plate variables have an angular frequency of ω. Thus their variation with time *t* is proportional to $e^{j\omega t}$. This factor will be omitted from the equations. A plane bending wave with transverse velocity amplitude v_{1+} is incident normally on the junction between the two plates (*z*-axis) from plate 1 (*x* is less than or equal to zero).

The ratios of the amplitudes of the propagating reflected wave, the nearfield reflected wave, the propagating transmitted wave and the nearfield transmitted waves to the amplitude of the forced incident waves, are r, r_j, t, t_j respectively. Also k_1 and k_2 are the freely bending propagating wave numbers of the two plates. The total velocities v_{y1} and v_{y2} in plates 1 and 2 are given by equation 6.14 of Cremer *et al*. (2005) as

$$v_{y1}(x) = v_{1+}\left(e^{-jk_1x} + re^{jk_1x} + r_j e^{k_1x}\right) \quad x \leq 0, \tag{4.1}$$

and

$$v_{y2}(x) = v_{1+}\left(te^{-jk_2x} + t_j e^{-k_2x}\right) \qquad x \geq 0. \tag{4.2}$$

At the junction of the plates ($y=0$, $x=0$), x is zero in equation (4.1) and (4.2) and the velocities of the two plates become

$$v_{y1}(0) = v_{1+}(1 + r + r_j) \quad x \le 0, \tag{4.3}$$

and

$$v_{y2}(0) = v_{1+}(t + t_j) \quad x \ge 0. \tag{4.4}$$

Because the boundary is pinned the velocities ($v_{y1}(0)$ and $v_{y2}(0)$) are equal to zero at x=0. Thus

$$1 + r + r_j = t + t_j = 0. \tag{4.5}$$

From Cremer *et al*. (2005) equation 3.69

$$w_z = \frac{\partial v_y}{\partial x}, \tag{4.6}$$

where w_z is the angular velocity of the plate about the z axis. By applying equation (4.6) to equations (4.1) and (4.2) we have the following equations

$$w_z = v_{1+}(-jk_1 e^{-jk_1 x} + jk_1 r e^{jk_1 x} + k_1 r_j e^{k_1 x}) \qquad x \le 0, \tag{4.7}$$

and

$$w_z = v_{1+}(-jk_2 t e^{-jk_2 x} - k_2 t_j e^{-k_2 x}) \quad x \ge 0. \tag{4.8}$$

At x=0, equations (4.7) and (4.8) become

$$w_z = v_{1+}(-jk_1 + jk_1 r + k_1 r_j), \tag{4.9}$$

and

$$w_z = v_{1+}(-jk_2 t - k_2 t_j). \tag{4.10}$$

Because the two plates are rigidly connected at $x=0$ the angular velocity given by equations (4.9) and (4.10) must be equal. Therefore

$$-jk_1 + jk_1 r + k_1 r_j = -jk_2 t - k_2 t_j. \tag{4.11}$$

The angular moment per unit length about the z axis is given by Cremer *et al*. (2005) equation 3.77

$$\frac{\partial M_z}{\partial t} = -B\frac{\partial w_z}{\partial x}. \tag{4.12}$$

Since M_z varies with time t as $e^{j\omega t}$, equation (4.12) gives the following equation for the angular moment

$$M_z = -\frac{B}{j\omega}\frac{\partial w_z}{\partial x}. \tag{4.13}$$

.

The plate 1 moment is given by

$$\begin{aligned} M_z &= -\frac{B}{j\omega}\frac{\partial w_z}{\partial x} \\ &= -\frac{B}{j\omega}v_{1+}(-k_1^2 e^{-jk_1 x} - k_1^2 r e^{jk_1 x} + k_1^2 r_j e^{k_1 x}) \qquad \text{when} \qquad x \le 0. \end{aligned} \tag{4.14}$$

Thus

$$M_z = v_{1+}\left(\frac{B_1 k_1^2 e^{-jk_1 x}}{j\omega} + \frac{B_1 k_1^2 r e^{jk_1 x}}{j\omega} - \frac{B_1 k_1^2 r_j e^{k_1 x}}{j\omega}\right) \qquad \text{when} \qquad x \le 0. \tag{4.15}$$

The plate 2 moment is given by

$$M_z = -\frac{B}{j\omega}\frac{\partial w_z}{\partial x} = -\frac{B}{j\omega}v_{1+}(-k_2^2 t e^{-jk_2 x} + k_2^2 t_j e^{-k_2 x}) \qquad \text{when} \qquad x \le 0. \tag{4.16}$$

Thus

$$M_z = v_{1+}\left(\frac{B_2 k_2^2 t e^{-jk_2 x}}{j\omega} - \frac{B_2 k_2^2 t_j e^{-k_2 x}}{j\omega}\right) \qquad \text{when} \qquad x \ge 0. \tag{4.17}$$

At the junction (x=0), the moments of plate 1 and 2, given by equations (4.15) and (4.17) are equal. Hence

$$v_{1+}\left(\frac{B_1 k_1^2}{j\omega} + \frac{B_1 k_1^2 r}{j\omega} - \frac{B_1 k_1^2 r_j}{j\omega}\right) = v_{1+}\left(\frac{B_2 k_2^2 t}{j\omega} - \frac{B_2 k_2^2 t_j}{j\omega}\right), \tag{4.18}$$

$$B_1k_1^2 r - B_1k_1^2 r_j - B_2k_2^2 t + B_2k_2^2 t_j = -B_1k_1^2. \tag{4.19}$$

$$r_j = (-1-r) \text{ and } t_j = -t. \tag{4.20}$$

Now using equation (4.20), r_j and t_j are replaced in equation (4.19) to give

$$B_1k_1^2 + 2B_1k_1^2 r - 2B_2k_2^2 t = -B_1k_1^2. \tag{4.21}$$

Divide equation (4.21) by $B_1k_1^2$.

$$1 + 2r - 2\frac{B_2k_2^2}{B_1k_1^2} t = -1. \tag{4.22}$$

Define

$$\psi = \frac{B_2k_2^2}{B_1k_1^2}. \tag{4.23}$$

.

Canceling out the 2's gives

$$r - \psi t = -1. \tag{4.24}$$

The above equation will be used as one of the simultaneous equations.

Rearranging equation (4.11) gives

$$jk_1 r + k_1 r_j + jk_2 t + k_2 t_j = jk_1. \tag{4.25}$$

Substituting equation (4.20) into the above equation (4.25) gives

$$jk_1 r - k_1 - k_1 r + jk_2 t - k_2 t = jk_1. \tag{4.26}$$

.

Dividing by k_1 gives

$$(j-1)r + (j\frac{k_2}{k_1} - \frac{k_2}{k_1})t = j+1. \tag{4.27}$$

κ is defined to be k_2/k_1

Thus equation (4.27) becomes

$$(-1+j)r+(-1+j)\kappa t=1+j. \tag{4.28}$$

Dividing (4.28) by $-1+j$ and multiplying the denominator and the numerator by the complex conjugate $-1-j$ gives

$$r+\kappa t=\frac{1+j}{-1+j}\frac{-1-j}{-1-j}. \tag{4.29}$$

and

$$r+\kappa t=-j. \tag{4.30}$$

Now there are four unknowns (t, t_j, r, r_j) and four equations to solve them. They will be solved using four simultaneous equations.

Equation (4.30) minus (4.24) is

$$\kappa t+\psi t=-j+1. \tag{4.31}$$

.

Solving for t, the transmitted wave is

$$t=\frac{1-j}{\kappa+\psi}, \tag{4.32}$$

and the transmitted nearfield is

$$t_j=\frac{-1+j}{\kappa+\psi}. \tag{4.33}$$

Rearranging equation (4.30) gives us the reflected wave

$$r=-j-\kappa t. \tag{4.34}$$

Substituting the value of t that has just been found in equation (4.32) into equation (4.34), gives

$$r=\frac{-j(\kappa+\psi)}{\kappa+\psi}-\frac{\kappa(1-j)}{\kappa+\psi}. \tag{4.35}$$

.

and

$$r = -\frac{\kappa + j\psi}{\kappa + \psi}. \tag{4.36}$$

Thus the reflected nearfield is

$$r_j = \frac{(-1+j)\psi}{\kappa + \psi}. \tag{4.37}$$

4.3 Derivation of wave numbers for the obliquely incident forced wave case.

In this section, the wave number equations ((6.142b), (6.142c), (6.142d)) from Cremer *et al*. (2005) are derived. Equation (6.142a) is also presented.

A single frequency, freely propagating bending wave in a plate satisfies the two dimensional homogeneous bending wave equation. From equation (3.184a) of Cremer *et al*. (2005), this is

$$\nabla^4 v_y - \frac{m\omega^2}{B} v_y = 0, \tag{4.38}$$

where the equation has been differentiated with respect to the time *t*. v_y is the transverse velocity of the plate in the direction of the *y* axis in figure 4.1. ω is the angular frequency, *m* is the mass per unit area and *B* is the bending stiffness of the plate.

Assume that there is a plane bending wave in the plate. Rotate the *x* and the *z* axes so that the *x*-axis points in the direction of propagation of the plate and so that the properties of the plane bending wave do not vary in the *z*-axis direction. Then equation (4.38) becomes

$$\frac{\partial^4}{\partial x^4} v_y - \frac{m\omega^2}{B} v_y = 0. \tag{4.39}$$

Let

$$v_y(x) = v_y e^{-jkx}, \tag{4.40}$$

where k is the wave number of the freely propagating plane bending wave in the plate. Then putting equation (4.40) into equation (4.39) gives

$$k^4 = \frac{m\omega^2}{B}. \tag{4.41}$$

Thus equation (4.38) can be written as

$$\nabla^4 v_y - k^4 v_y = 0. \tag{4.42}$$

Since, in Cartesian co-ordinates

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}, \tag{4.43}$$

equation (4.42) can be written as

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)^2 v_y - k^4 v_y = 0, \tag{4.44}$$

or

$$\frac{\partial^4 v_y}{\partial x^4} + 2\frac{\partial^4 v_y}{\partial x^2 \partial z^2} + \frac{\partial^4 v_y}{\partial z^4} - k^4 v_y = 0. \tag{4.45}$$

Equation (4.40) become

$$v_y(\mathrm{x}) = v_y e^{-j\underline{\mathrm{k}}.\underline{\mathrm{x}}}, \tag{4.46}$$

where,

$$\underline{\mathrm{k}} = (k_x, k_z), \tag{4.47}$$

and

$$\underline{\mathrm{x}} = (x, z). \tag{4.48}$$

Equation (4.46) can be written as

$$v_y(x,z) = v_y e^{-j(k_x x + k_z z)}. \tag{4.49}$$

Putting equation (4.49) into equation (4.45) gives

$$k_x^4 + 2k_x^2 k_z^2 + k_z^4 - k^4 = 0, \tag{4.50}$$

or

$$\left(k_x^2 + k^2{}_z\right)^2 = k^4. \tag{4.51}$$

Therefore

$$k_x^2 + k_z^2 = \pm k^2, \tag{4.52}$$

Thus

$$k_x = \pm\sqrt{\pm k^2 - k_z^2}. \tag{4.53}$$

There are four possible values of k_x.

$$k_x = \pm\sqrt{k^2 - k_z^2}, \tag{4.54}$$

and

$$k_x = \pm j\sqrt{k^2 + k_z^2}. \tag{4.55}$$

For a plane wave which is obliquely incident on the junction of the two plates at an angle of θ_i relative to the positive direction of the positive x-axis from plate 1, with forced wave number k_i,

$$k_{zi} = k_i \sin\theta_i. \tag{4.56}$$

Also

$$k_{xi} = k_i \cos\theta_i = \chi_a k_1 \cos\theta_i, \tag{4.57}$$

where

$$\chi_a = \frac{k_i}{k_1}. \tag{4.58}$$

In order to maintain continuity of angular velocity and angular momentum at the junction between the two plates, all reflected and transmitted waves must have the same variability in the z-axis direction as the incidence wave, namely

$$E(z) = e^{-jk_{zi}} = e^{-jk_i \sin\theta_i}. \tag{4.59}$$

Thus

$$k_{z2} = k_{z1} = k_z = k_i \sin\theta_i, \tag{4.60}$$

where *2*, *1* and *i* denote the transmitted, reflected and incidence waves.

Since the reflected waves propagate or decay in the negative x direction, the propagating reflected wave has a k_x value given by

$$-k_x = k_{x1} = +\sqrt{k_1^2 - k_i^2 \sin^2\theta_i} = k_1\sqrt{1-\chi_a^2 \sin^2\theta_i}. \tag{4.61}$$

Note that this is only a propagating wave if

$$k_1 > k_i|\sin\theta_i|. \tag{4.62}$$

If

$$k_1 < k_i|\sin\theta_i|, \tag{4.63}$$

the reflected wave becomes a non-propagating near field wave.

If the incidence wave is a freely propagating wave

$$k_i = k_1 = k_r. \tag{4.64}$$

And θ_i will be denoted by θ_1.

Equation (4.60) gives

$$k_{z2} = k_{z1} = k_z = k_1 \sin\theta_1, \tag{4.65}$$

and equation (4.61) gives

$$-k_x = k_{x1} = +k_1\sqrt{1-\sin^2\theta_1} = +k_1\cos\theta_1. \tag{4.66}$$

Thus

$$-jk_x = jk_{x1} = jk_1\cos\theta_1. \tag{4.67}$$

Denoting the angle of reflection relative to the direction of the positive x-axis by θ_r gives

$$\cos\theta_r = \frac{k_x}{k_1} = -\cos\theta_1, \tag{4.68}$$

and

$$\sin\theta_r = \frac{k_{z1}}{k_1} = \sin\theta_1. \tag{4.69}$$

Hence

$$\theta_r = \pi - \theta_1. \tag{4.70}$$

Notice that the angle is in radians for the case of a freely propagating incident wave.

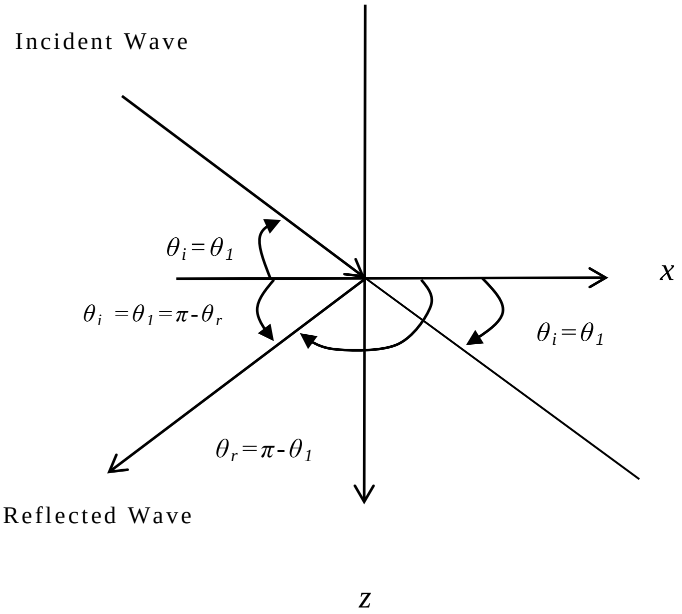

Figure 4.2 The angle of reflection equals the angle of incidence for a freely propagating wave. The positive y-axis points vertically out of the page.

Figure 4.2 shows that the reflected angle defined in the usual way is equal to the incident angle if the incident wave is a freely propagating one.

The reflected decaying nearfield sound field has

$$k_x = j\sqrt{k_1^2 + k_{zi}^2}, \tag{4.71}$$

and

$$\begin{aligned} k_{N1} &= -jk_x \\ &= k_1\sqrt{1 + \chi_a^2 \sin^2\theta_i}, \end{aligned} \tag{4.72}$$

where $\chi_a = \dfrac{k_i}{k_1}$. If the incident wave is freely propagating

$$k_{N1} = k_1\sqrt{1 + \sin^2\theta_1}, \tag{4.73}$$

This is equation (6.142c) of Cremer *et al*. (2005).

The propagating transmitted wave has

$$k_x = k_{x2} = \sqrt{k_2^2 - k_{zt}^2}$$
$$= k_1\sqrt{\kappa^2 - \chi_a^2 \sin^2\theta_i}\,, \tag{4.74}$$

where $\kappa = \dfrac{k_2}{k_1}$. This is equation (6.142b) of Cremer *et al*. (2005). If the incident wave is freely propagating,

$$\chi_a = 1, \quad \theta_{\mathrm{i}} = \theta_1 \quad \text{and} \quad k_{x2} = k_1\sqrt{\kappa^2 - \sin^2\theta_1}\,. \tag{4.75}$$

Similarly to the reflected wave, this transmitted wave is only a propagating wave if

$$k_2 > k_i\left|\sin\theta_i\right|. \tag{4.76}$$

If

$$k_2 < k_i\left|\sin\theta_i\right|, \tag{4.77}$$

total internal reflection occurs and the transmitted wave becomes a non-propagating near field wave.

If the incident wave is a freely propagating wave and if the transmitted freely propagating wave travels at an angle of θ_2 , relative to the direction of the positive x-axis,

$$k_{z2} = k_2\sin\theta_2 = k_{z1} = k_1\sin\theta_1. \tag{4.78}$$

This is Snells law and equation (6.142a) of Cremer *et al*. (2005). The transmitted decaying nearfield wave has

$$k_x = -j\sqrt{k_2^2 + k_{z^2}^2}\,, \tag{4.79}$$

and

$$k_{N2} = jk_x$$
$$= k_1\sqrt{\kappa^2 + \chi_a^2\sin^2\theta_i}\,. \tag{4.80}$$

If the incident wave is freely propagating,

$$k_{N2} = k_1\sqrt{\kappa^2 + \sin^2\theta_1}, \tag{4.81}$$

This is equation (6.142d) of Cremer *et al*. (2005).

4.4 The angular velocity and the torsional moment for the obliquely incident wave case.

The transverse bending wave velocities in plates 1 and 2 are

$$v_{y1} = v_{1+}E(z)\left(e^{-jk_{x1}x} + re^{jk_{x1}x} + r_j e^{k_{N1}x}\right) \; x \le 0, \tag{4.82}$$

and

$$v_{y2} = v_{1+}E(z)\left(te^{-jk_{x2}x} + te^{-k_{N2}x}\right), \tag{4.83}$$

where v_{1+} is the rms velocity of the forced incident wave. At x=0, equations (4.82) and (4.83) are zero because of the pinning.

$$1 + r + r_j = 0, \tag{4.84}$$

and

$$t + t_j = 0. \tag{4.85}$$

The angular velocities about the z-axis in plates 1 and 2 are

$$w_z = \frac{\partial v_y}{\partial x}, \tag{4.86}$$

as given in equation (4.6) and in equation (3.69) of Cremer *et al*. (2005).

Thus

$$w_{z1} = v_{1+}E(z)\left(-jk_{xi}e^{-jk_{x1}x} + jk_{xr}re^{jk_{x1}x} + k_{Nr}r_j e^{k_{N1}x}\right) \quad x \le 0, \tag{4.87}$$

and

$$w_{z2} = v_{1+}E(z)\left(-jk_{x2}te^{-jk_{x2}x} - k_{N2}t_j e^{-k_{N2}x}\right) \quad x \ge 0. \tag{4.88}$$

At x=0, equations (4.87) and (4.88) are equal to each other because of continuity.

Thus

$$-jk_{x1}+jk_{x1}r+k_{N1}r_j=-jk_{x2}t-k_{N2}t_j. \tag{4.89}$$

This is equation (4.11), with the first k_1 replaced with k_{x1} , the second k_1 replaced with k_{x1}, the third k_1 replaced with k_{N1}, the first k_2 replaced with k_{x2} and the second k_2 replaced with k_{N2}.

The angular velocity about the x-axis is

$$w_x=-\frac{\partial v_y}{\partial z}. \tag{4.90}$$

Cremer *et al*. (2005) points out in section 6.7.2.2 that this gives

$$w_x = jk_z v_y \tag{4.91}$$

Equation (4.91) means that the angular velocity about the x-axis is completely determined by the values of the other variables.

Since all the k_z's are the same on both sides of the z-axis junction and the velocities v_y on each side of the junction are equal at the junction, the w_x's are also equal on both sides of the junction (where $x=0$ and $y=0$).

Equation (4.13) of the previous section can be written as

$$M_z=-\frac{B}{j\omega}\frac{\partial^2 v_y}{\partial x^2}. \tag{4.92}$$

For an obliquely incident wave this has to be modified to (see Cremer *et al*. (2005) equation (6.144b)).

$$M_{xz}=-\frac{B}{j\omega}\left(\frac{\partial v_y}{\partial x^2}+\mu\frac{\partial^2 v_y}{\partial z^2}\right), \tag{4.93}$$

where μ is Poisson's ratio. The term containing it appears because the Poisson expansion and contraction in the x -axis direction due to wave propagation in the z-axis direction generates a moment about the z-axis.

Equation (4.93) becomes

$$M_{xz}=-\frac{B}{j\omega}\left(\frac{\partial^2 v_y}{\partial x^2}-\mu k_z^2 v_y\right). \tag{4.94}$$

For the pinned joint case being considered here, when x=0, v_y=0. Thus equation (4.94) reduces to

$$M_{xz}(0) = -\frac{B}{j\omega}\frac{\partial^2 v_y}{\partial x^2}(0) = -\frac{B}{j\omega}\frac{\partial w_z}{\partial x}(0). \tag{4.95}$$

Because of continuity of M_{xz} at $x=0$,

$$B_1 \frac{\partial w_{z1}}{\partial x}(0) = B_2 \frac{\partial w_{z2}}{\partial x}(0). \tag{4.96}$$

Differentiating equations (4.87) and (4.88) with respect to x gives

$$B_1 \frac{\partial w_{z1}}{\partial x} = B_1 v_{1+} E(z)\left(-k_{xi}^2 e^{-jk_{xi}x} - k_{x1}^2 r e^{jk_{x1}x} + k_{N1}^2 r_j e^{k_{N1}x}\right) \quad x \le 0, \tag{4.97}$$

and

$$B_2 \frac{\partial w_{z2}}{\partial x} = B_2 v_{1+} E(z)\left(-k_{x2}^2 t e^{-jk_{x2}t} + k_{N2}^2 t_j e^{-k_{N2}x}\right) \quad x \ge 0. \tag{4.98}$$

Combining equations (4.97) to (4.98) gives

$$B_1\left(-k_{xi}^2 - k_{x1}^2 r + k_{N1}^2 r_j\right) = B_2\left(-k_{x2}^2 t + k_{N2} t_j\right). \tag{4.99}$$

Cremer *et al*. (2005) equation (6.144a) gives the torsional moment per unit length as

$$M_{xx} = \frac{B(1-\mu)}{j\omega}\frac{\partial^2 v}{\partial x \partial z} = -\frac{B(1-\mu)}{\omega} k_z \frac{\partial v_y}{\partial x} = -\frac{B(1-\mu)}{\omega} k_z \omega_z. \tag{4.100}$$

This shows that the torsional moment per unit length is completely determined by the values of the other variables.

Although not used here because the joint is pinned, it is worthwhile noting that equations, (3.148e) and (6.144c) of Cremer *et al*. (2005) give the shear force per unit length acting on a plane normal to the x-axis as

$$\begin{aligned} Q_x &= -\frac{\partial M_{xz}}{\partial x} - \frac{\partial M_{zz}}{\partial z} \\ &= \frac{B}{j\omega}\left(\frac{\partial^3 v_y}{\partial x^3} - k_z^2 w_z\right) \end{aligned} \tag{4.101}$$

Because of the appearance of the third partial derivative of v_y with respect to x, Q_x is independent of the other variables.

From equation (6.145) of Cremer *et al*. (2005), the external supporting or reaction force per unit length at a boundary is

$$F_x = Q + \frac{\partial M_{xx}}{\partial z} = \frac{B}{j\omega}\left(\frac{\partial^3 v_y}{\partial x^3} + (2-\mu)\frac{\partial^3 v_y}{\partial x \partial z^2}\right) = \frac{B}{j\omega}\left(\frac{\partial^3 v_y}{\partial x^3} - (2-\mu)k_z^2 w_z\right). \tag{4.102}$$

Note that Cremer *et al*. (2005) have the wrong sign in front of the partial derivative of the torsional moment M_{xx} and differentiate it with respect to x, rather than z as in their earlier editions.

4.5 Derivation of the transmitted and reflected waves for obliquely incident freely propagating waves.

If the incident wave is freely propagating, $k_{xi}=k_{x1}$, and $\theta_i=\theta_1$. Equations (4.89) and (4.99) become

$$-jk_{x1}(1-r) + k_{N1}r_j = -jk_{x2}t - k_{N2}t_j, \tag{4.103}$$

and

$$B_1\left[-k_{x1}^2(1+r) + k_{N1}^2 r_j\right] = B_2\left[-k_{x2}^2 t + k_{N2}^2 t_j\right]. \tag{4.104}$$

Equations (4.84), (4.85), (4.103) and (4.104) agree with equation (6.146) of Cremer *et al*. (2005).

Using equations (4.84), (4.85) in equations (4.103) and (4.104) gives

$$(jk_{x1} - k_{N1})r + (jk_{x2} - k_{N2})t = jk_{x1} + k_{N1}, \tag{4.105}$$

and

$$B_1\left(k_{x1}^2 + k_{N1}^2\right)r - B_2\left(k_{x2}^2 + k_{N2}^2\right)t = -B_1\left(k_{x1}^2 + k_{N1}^2\right). \tag{4.106}$$

From equations (4.66) and (4.73).

$$k_{x1}^2 + k_{N1}^2 = k_1^2\left(\cos^2\theta_1 + 1 + \sin^2\theta_1\right) = 2k_1^2. \tag{4.107}$$

From equations (4.75) and (4.81)

$$k_{x2}^2 + k_{N2}^2 = k_1^2\left(\kappa^2 - \sin^2\theta_1 + \kappa^2 + \sin^2\theta_1\right) = 2\kappa^2 k_1^2. \tag{4.108}$$

Inserting equations (4.107) and (4.108) into equation (4.106) gives

$$r - \psi t = -1, \tag{4.109}$$

where

$$\psi = \frac{B_2}{B_1}\kappa^2 = \frac{B_2 k_2^2}{B_1 k_1^2}. \tag{4.110}$$

Note that equation (4.109) is the same as equation (4.24).

Using equation (4.109) to replace r in equation (4.105) gives

$$t = \frac{2jk_{x1}}{j\left(k_{x2} + \psi k_{x1}\right) - \left(k_{N2} + \psi k_{N1}\right)}. \tag{4.111}$$

Also

$$r = \frac{j\left(\psi k_{x1} - k_{x2}\right) + \left(k_{N2} + \psi k_{N1}\right)}{j\left(k_2 + \psi k_{x1}\right) - \left(k_{N2} + \psi k_{N1}\right)}. \tag{4.112}$$

Substituting equation (4.109) into equation (4.84) gives

$$r_j = -\psi t. \tag{4.113}$$

Combining equations (4.111) and (4.113) gives

$$r_j = \frac{-2j\psi k_{x1}}{j\left(k_{x2} + \psi k_{x1}\right) - \left(k_{N2} + \psi k_{N1}\right)}. \tag{4.114}$$

Combining equations (4.85) and (4.111) gives

$$t_j = \frac{-2jk_{x1}}{j\left(k_{x2} + \psi k_{x1}\right) - \left(k_{N2} + \psi k_{N1}\right)}. \tag{4.115}$$

$$t = \frac{2j\cos\theta_1}{j(\sqrt{\kappa^2 - \sin^2\theta_1} + \psi\cos\theta_1) - \psi\sqrt{1 + \sin^2\theta_1} - \sqrt{\kappa^2 + \sin^2\theta_1}} \tag{4.116}$$

This equation is the same as equation (6.147a) of Cremer *et al*. (2005) apart from a difference in sign which arises from the fact that in plate 2, the positive *y*-axis direction of this thesis is transformed to the negative x-axis direction in Cremer *et al.* (2005) because this thesis considers a straight pinned junction while Cremer *et al.* (2005) consider a right angle pinned junction.

$$r = \frac{\psi j\cos\theta_1 - j\sqrt{\kappa^2 - \sin^2\theta_1} + \psi\sqrt{1+\sin^2\theta_1} + \sqrt{\kappa^2 + \sin^2\theta_1}}{j(\sqrt{\kappa^2 - \sin^2\theta_1} + \psi\cos\theta_1) - \psi\sqrt{1+\sin^2\theta_1} - \sqrt{\kappa^2 + \sin^2\theta_1}}. \quad (4.117)$$

Again this is the same as equation (6.147b) of Cremer *et al.* (2005).

$$r_j = \frac{-2\psi j\cos\theta_1}{j(\sqrt{\kappa^2 - \sin^2\theta_1} + \psi\cos\theta_1) - \psi\sqrt{1+\sin^2\theta_1} - \sqrt{\kappa^2 + \sin^2\theta_1}}. \quad (4.118)$$

$$t_j = \frac{-2j\cos\theta_1}{j(\sqrt{\kappa^2 - \sin^2\theta_1} + \psi\cos\theta_1) - \psi\sqrt{1+\sin^2\theta_1} - \sqrt{\kappa^2 + \sin^2\theta_1}}. \quad (4.119)$$

4.6 Obliquely incident forced waves

Substituting equations (4.66), (4.73), (4.75) and (4.81) into equations (4.111), (4.112), and (4.114), gives the following equations. Putting θ_1 equal to zero reduces these equations to the equations (4.32), (4.33), (4.36) and (4.37).

The obliquely incident wave is assumed to be a forced wave with a wave number of k_i. Using equations (4.84) and (4.85) in equations (4.89) gives

$$\left(jk_{x1} - k_{N1}\right)r + \left(jk_{x2} - k_{N2}\right)t = jk_{xi} + k_{N1}, \quad (4.120)$$

and

$$B_1\left(k_{x1}^2 + k_{N1}^2\right)r - B_2\left(k_{x2}^2 + k_{N2}^2\right)t = -B_1\left(k_{xi}^2 + k_{N1}^2\right). \quad (4.121)$$

From equations (4.61) and (4.72)

$$k_{x1}^2 + k_{N1}^2 = k_1^2 - k_i^2 \sin^2\theta_i + k_1^2 + k_i^2 \sin^2\theta_i = 2k_1^2. \tag{4.122}$$

From equations (4.74) and (4.80)

$$k_{x2}^2 + k_{N2}^2 = k_2^2 - k_{z2}^2 + k_2^2 + k_{z2}^2 = 2k_2^2 = 2\kappa^2 k_1^2, \tag{4.123}$$

where $\kappa = \dfrac{k_2}{k_1}$.

From equations (4.57) and (4.72)

$$\begin{aligned} k_{xi}^2 + k_{N1}^2 &= k_i^2 \cos\theta_i^2 + k_1^2 + k_i^2 \sin^2\theta_i \\ &= k_1^2\left(1+\chi_a^2\right). \end{aligned} \tag{4.124}$$

Using equations (4.122), (4.123) and (4.124) in equation (4.121) gives

$$r - \psi t = \frac{-\left(1+\chi_a^2\right)}{2}. \tag{4.125}$$

When the incident wave is freely propagating $\chi_a = 1$, and equation (4.125) agrees with equation (4.109).

Substituting equation (4.125) into equation (4.120) gives

$$t = \frac{\left(jk_{xi} + k_{N1}\right) + \left(jk_{x1} - k_{N1}\right)\left(1+\chi_a^2\right)/2}{\psi\left(jk_{x1} - k_{N1}\right) + \left(jk_{x2} - k_{N2}\right)}. \tag{4.126}$$

Substituting equation (4.126) into equation (4.125) gives

$$r = \frac{\psi\left(jk_{xi} + k_{N1}\right) - \left(jk_{x2} - k_{N2}\right)\left(1+\chi_a^2\right)/2}{\psi\left(jk_{x1} - k_{N1}\right) + \left(jk_{x2} - k_{N2}\right)}. \tag{4.127}$$

From equations (4.85) and (4.126)

$$t_j = -\frac{\left(jk_{xi} + k_{N1}\right) + \left(jk_{x1} - k_{N1}\right)\left(1+\chi_a^2\right)/2}{\psi\left(jk_{x1} - k_{N1}\right) + \left(jk_{x2} - k_{N2}\right)}. \tag{4.128}$$

From equations (4.84) and (4.127)

$$r_j = \frac{-j\psi\left(k_{xi} + k_{x1}\right) + \left(jk_{x2} - k_{N2}\right)\left(\chi_a^2 - 1\right)}{\psi\left(jk_{x1} - k_{N1}\right) + \left(jk_{x2} - k_{N2}\right)}. \tag{4.129}$$

Substituting equations (4.56), (4.61), (4.72), (4.74), and (4.80) into equations (4.126), (4.127), (4.128), and (4.129) gives

$$t = \frac{\begin{array}{c}\left(j\chi_a \cos\theta_i + \sqrt{1+\chi_a^2 \sin^2\theta_i}\right) \\ + \left(j\sqrt{1-\chi_a^2 \sin^2\theta_i} - \sqrt{1+\chi_a^2 \sin^2\theta_i}\right)\left(\chi_a^2 + 1\right)/2\end{array}}{\begin{array}{c}\psi\left(j\sqrt{1-\chi_a^2 \sin^2\theta_i} - \sqrt{1+\chi_a^2 \sin^2\theta_i}\right) \\ + \left(j\sqrt{\kappa^2 - \chi_a^2 \sin^2\theta_i} - \sqrt{\kappa^2 + \chi_a^2 \sin^2\theta_i}\right)\end{array}}, \tag{4.130}$$

and

$$r = \frac{\left[\begin{array}{l}\psi\left(j\chi_a \cos\theta_i + \sqrt{1+\chi_a^2 \sin^2\theta_i}\right) \\ - \left(j\sqrt{\kappa^2 - \chi_a^2 \sin^2\theta_i} - \sqrt{\kappa^2 + \chi_a^2 \sin^2\theta_i}\right)\left(\chi_a^2 + 1\right)/2\end{array}\right]}{\begin{array}{c}\psi\left(j\sqrt{1-\chi_a^2 \sin^2\theta_i} - \sqrt{1+\chi_a^2 \sin^2\theta_i}\right) \\ + \left(j\sqrt{\kappa^2 - \chi_a^2 \sin^2\theta_i} - \sqrt{\kappa^2 + \chi_a^2 \sin^2\theta_i}\right)\end{array}}, \tag{4.131}$$

and

$$t_j = -\frac{\begin{array}{c}\left(j\chi_a \cos\theta_i + \sqrt{1+\chi_a^2 \sin^2\theta_i}\right) \\ + \left(j\sqrt{1-\chi_a^2 \sin^2\theta_i} - \sqrt{1+\chi_a^2 \sin^2\theta_i}\right)\left(\chi_a^2 + 1\right)/2\end{array}}{\begin{array}{c}\psi\left(j\sqrt{1-\chi_a^2 \sin^2\theta_i} - \sqrt{1+\chi_a^2 \sin^2\theta_i}\right) \\ + \left(j\sqrt{\kappa^2 - \chi_a^2 \sin^2\theta_i} - \sqrt{\kappa^2 + \chi_a^2 \sin^2\theta_i}\right)\end{array}}, \tag{4.132}$$

and

$$r_j = \frac{\left[\begin{array}{l}- j\psi\left(\chi_a \cos\theta_i + \sqrt{1-\chi_a^2 \sin^2\theta_i}\right) \\ + \left(j\sqrt{\kappa^2 - \chi_a^2 \sin^2\theta_i} - \sqrt{\kappa^2 + \chi_a^2 \sin^2\theta_i}\right)\left(\chi_a^2 - 1\right)/2\end{array}\right]}{\begin{array}{c}\psi\left(j\sqrt{1-\chi_a^2 \sin^2\theta_i} - \sqrt{1+\chi_a^2 \sin^2\theta_i}\right) \\ + \left(j\sqrt{\kappa^2 - \chi_a^2 \sin^2\theta_i} - \sqrt{\kappa^2 + \chi_a^2 \sin^2\theta_i}\right)\end{array}}, \tag{4.133}$$

Let

$$s^2 = \chi_a^2 \sin^2\theta_i = \sin^2\theta_1 = \kappa^2 \sin^2\theta_2. \tag{4.134}$$

Then equations (4.130), (4.131), (4.132), and (4.133) can be written as

$$t = \frac{\left(j\sqrt{\chi_a^2 - s^2} + \sqrt{1+s^2}\right) + \left(j\sqrt{1-s^2} - \sqrt{1+s^2}\right)\left(\chi_a^2 + 1\right)/2}{\Delta}, \tag{4.135}$$

$$r = \frac{\psi\left(j\sqrt{\chi_a^2 - s^2} + \sqrt{1+s^2}\right) + \left(j\sqrt{\kappa^2 - s^2} - \sqrt{\kappa^2 + s^2}\right)\left(\chi_a^2 + 1\right)/2}{\Delta}, \tag{4.136}$$

$$t_j = -\frac{\left(j\sqrt{\chi_a^2 - s^2} + \sqrt{1+s^2}\right) + \left(j\sqrt{1-s^2} - \sqrt{1+s^2}\right)\left(\chi_a^2 + 1\right)/2}{\Delta}, \tag{4.137}$$

and

$$r_j = \frac{-j\psi\left(\sqrt{\chi_a^2 - s^2} + \sqrt{1+s^2}\right) + \left(j\sqrt{\kappa^2 - s^2} - \sqrt{\kappa^2 + s^2}\right)\left(\chi_a^2 - 1\right)/2}{\Delta}, \tag{4.138}$$

where

$$\Delta = \psi\left(j\sqrt{1-s^2} - \sqrt{1+s^2}\right) + \left(j\sqrt{\kappa^2 - s^2} - \sqrt{\kappa^2 + s^2}\right). \tag{4.139}$$

Equations (4.126), (4.130) and (4.135) agree with equation (9) of Villot and Guigou-Carter (2000) apart from differing in sign. This difference in sign is due to the fact that the two plates are in the same plane in this thesis, while in Villot and Guigou-Carter (2000) the second plate has been rotated through +90 degrees in order to form a right angled junction. Thus the velocity in the positive *y*-axis direction in the second plate becomes a velocity in the negative *x*-axis direction in the Villot and Guigou-Carter (2000) paper. Note that the positive direction of the *x*-axis in Villot and Guigou-Carter (2000) figure 6a is incorrect. It does not correspond to the signs of the exponential exponents in their equation (4). Also note that once the positive direction of the *x*-axis is changed in their figure 6, the positive direction of the *z*-axis also has to be changed in order to maintain a right handed co-ordinate system.

Equation (4.135) can be written as

$$t = \frac{-\sqrt{1+s^2}\left(\chi_a^2 - 1\right)/2 + j\left[\sqrt{\chi_a^2 - s^2} + \sqrt{1-s^2}\left(\chi_a^2 + 1\right)/2\right]}{-\left(\psi\sqrt{1+s^2} + \sqrt{\kappa^2 + s^2}\right) + j\left(\psi\sqrt{1-s^2} + \sqrt{\kappa^2 - s^2}\right)}. \tag{4.140}$$

4.7 Transmitted bending wave power.

As was shown in chapters 2 and 3 for the acoustical case, it is not possible to calculate the net intensity incident on a junction by separately calculating the intensity of the incident wave and the intensity of the reflected wave, when the incident wave is not freely propagating. This is because the cross terms in the intensity calculation do not cancel unless the incident wave is freely propagating. Thus only the total intensity incident on the junction can be calculated. Because this is equal to the transmitted intensity by energy conservation, only the transmitted intensity will be calculated in this chapter. One of the consequences of not being able to calculate a meaningful incident intensity due to a forced incident wave alone, is that it is not possible to meaningfully define a transmission efficiency as Villot and Guigou-Carter (2000) have attempted to do in their equation (10). This is one of their two errors which are corrected in this thesis.

The transmitted bending wave power per unit length of the plate junction I is now derived. From equation (4.88) the angular velocity about the z-axis in plate 2 is

$$w_{z2} = v_{1+}E(z)\left(-jk_{x2}te^{-jk_{x2}x} - k_{N2}t_j e^{-k_{N2}x}\right). \tag{4.141}$$

Now $t_j = -t$

Thus

$$w_{z2} = v_{1+}E(z)t\left(-jk_{x2}e^{-jk_x x} - k_{N2}e^{-k_{N2}x}\right). \tag{4.142}$$

At the junction, $x=0$, thus,

$$w_{z2}(x=0) = v_{1+}E(z)t\left(-jk_{x2} + k_{N2}\right). \tag{4.143}$$

From equation (4.94), the moment per unit length about the z-axis, M_{xz2}, which is exerted across a line normal to the x-axis in plate 2 is

$$M_{xz2} = -\frac{B_2}{j\omega}\left(\frac{\partial^2 v_y}{\partial x^2} - \mu k_z^2 v_y\right). \tag{4.144}$$

But the transverse velocity $v_y\ (x=0)$ is zero because of the pinned joint. Thus

$$M_{xz2}(x=0) = -\frac{B_2}{j\omega}\left(\frac{\partial w_{z2}}{\partial x}(x=0)\right) = j\frac{B_2}{\omega}\frac{\partial w_{z2}}{\partial x}(x=0). \tag{4.145}$$

Differentiating equation (4.142), setting $x=0$ and inserting into equation (4.145) , gives

$$M_{xz2}(x=0)= j\frac{B_2}{\omega}v_{1+}E(z)t\left[-k_{x2}^2-k_{N2}^2\right]=-j\frac{B_2}{\omega}v_{1+}E(z)t\left(k_{x2}^2+k_{N2}^2\right). \qquad (4.146)$$

The transmitted bending wave power (I) per unit length of the plate junction is the real part of the product of the angular velocity and the moment because the transverse velocity of the junction is zero.

$$\begin{aligned} I &= \mathrm{Re}\left(\omega_{z2}(x=0)M_{xz2}(x=0)\right) \\ &= \left|v_{1+}\right|^2\left|t^2\right|\mathrm{Re}\left[k_{x2}^{*}\left\{(k_{x2})^2+k_{N2}^2\right\}\frac{B_2}{\omega}-j\frac{B_2}{\omega}k_{N2}\left((k_{x2})^2+k_{N2}^2\right)\right]. \end{aligned} \qquad (4.147)$$

Since $k_{x2}^2 = k_2^2 - k_i^2\sin^2\theta_i$, k_{x2}^2 is always real. Thus

$$I = \left|v_{1+}\right|^2\left|t\right|^2\left[k_{x2}^2+k_{N2}^2\right]\frac{B_2}{\omega}\mathrm{Re}\left[k_{x2}\right]. \qquad (4.148)$$

Now

$$k_{x2}^2 = k_2^2 - k_i^2\sin^2\theta_i, \qquad (4.149)$$

and

$$k_{N2}^2 = k_2^2 + k_i^2\sin^2\theta_i. \qquad (4.150)$$

Thus

$$k_{x2}^2 + k_{N2}^2 = 2k_2^2. \qquad (4.151)$$

From equation (4.74)

$$\begin{aligned} k_{x2} &= k_2\sqrt{1-\left(\frac{k_i}{k_2}\right)^2\sin^2\theta_i} \\ &= k_2\sqrt{1-\left(\frac{\chi_a}{\kappa}\right)^2\sin^2\theta_i} \end{aligned} \qquad (4.152)$$

Inserting equations (4.151) and (4.152) into equation (4.148) gives

$$I = 2\left|v_{1+}\right|^2\left|t\right|^2\frac{B_2k_2^3}{\omega}\mathrm{Re}\left(\sqrt{1-\left(\frac{\chi_a}{\kappa}\right)^2\sin^2\theta_i}\right). \qquad (4.153)$$

Now from the homogeneous bending wave equation (see equation (4.41)),

$$\frac{B_2 k_2^4}{m_2} = \omega^2. \tag{4.154}$$

Therefore

$$\frac{B_2 k_2^3}{\omega} = \frac{m_2 \omega}{k_2}. \tag{4.155}$$

Since

$$c_2 = \frac{\omega}{k_2}, \tag{4.156}$$

the following equation is obtained.

$$\frac{B_2 k_2^3}{\omega} = m_2 c_2. \tag{4.157}$$

Putting equation (4.157) into (4.153) gives

$$I = 2|v_{1+}|^2 |t|^2 m_2 c_2 \operatorname{Re}\left(\sqrt{1 - \left(\frac{\chi_a}{\kappa}\right)^2 \sin^2 \theta_i}\right). \tag{4.158}$$

Hence

$$I = 2|v_{1+}|^2 |t|^2 m_2 c_2 \operatorname{Re}\left(\sqrt{1 - \frac{s^2}{\kappa^2}}\right), \tag{4.159}$$

and

$$\frac{I}{2|v_{1+}|^2 m_2 c_2} = |t|^2 \operatorname{Re}\left(\sqrt{1 - \frac{s^2}{\kappa^2}}\right), \tag{4.160}$$

The transmitted bending wave power (I) per unit length of the plate junction can also be derived from equations (3.83), (3.84) and (3.92) of Cremer *et al*. (2005).

$$\begin{aligned} I &= 2m_2 c_2 |v_{1+}|^2 |t|^2 \operatorname{Re}(\cos\theta_2) \\ &= 2m_2 c_2 |v_{1+}|^2 |t|^2 \operatorname{Re}\left(\sqrt{1 - \frac{s^2}{\kappa}}\right) \end{aligned} \tag{4.161}$$

where m_2 is the mass of plate 2, c_2 is the bending wave speed in plate 2 and $|v_{1+}|$ is the root mean square velocity of the forced incident bending wave. Note that the factor of

2 does not appear in Cremer *et al.*(2005) because they are using peak wave amplitude instead of root mean square (rms) amplitude as is being used here.

4.8 The power per unit length transmitted by a diffuse bending wave field.

The power per unit length transmitted to plate 2 by a diffuse bending wave field in plate 1 will now be calculated. This power is proportional to the integral of equation (4.161) over the incident angle θ_i from 0 radians to $\pi/2$ radians or the minimum angle at which total internal reflection occurs when the real function in equation (4.161) is equal to zero. This occurs when

$$1-\frac{s^2}{\kappa^2}\leq 0. \tag{4.162}$$

This means that

$$\chi_a^2\sin^2\theta_i = s^2 \geq \kappa^2. \tag{4.163}$$

Rearranging gives,

$$\left|\sin\theta_i\right| \geq \frac{\kappa}{\chi_a}, \tag{4.164}$$

and

$$\theta_i \geq \arcsin\left(\frac{\kappa}{\chi_a}\right). \tag{4.165}$$

Thus the upper limit of the integral is

$$\theta_u = \begin{cases} \dfrac{\pi}{2} & \text{if} \quad \kappa \geq \chi_a \\ \arcsin\left(\dfrac{\kappa}{\chi_a}\right) & \text{if} \quad \kappa < \chi_a \end{cases}. \tag{4.166}$$

Hence the bending wave intensity transmitted by an incident diffuse forced bending wave field in plate 1 is proportional to

$$\int_0^{\theta_u} |t|^2 \operatorname{Re}\left(\sqrt{1-\frac{s^2}{\kappa^2}} \right) d\theta. \tag{4.167}$$

4.9 The transmitted power when plate 1 is excited by a diffuse acoustic field.

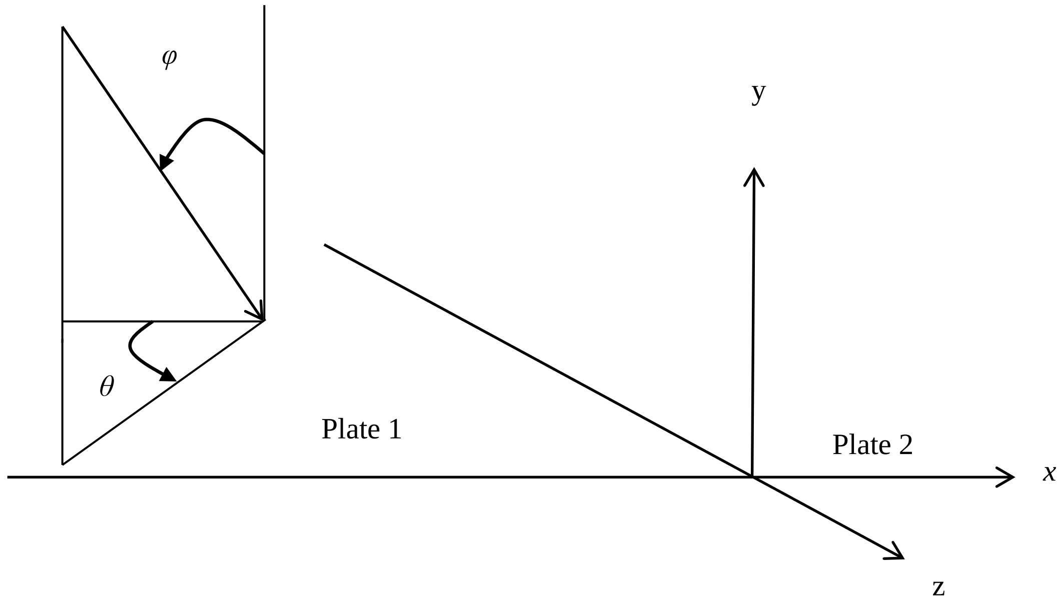

Figure 4.3 An incident acoustic sound wave.

From equation (12) of Villot and Guigou-Carter (2000), the wave impedance of plate 1 is

$$z = \frac{B_1}{j\omega}\left(k_i^2 - k_1^4\right) + B_1\eta_1 k_1^4. \tag{4.168}$$

where B_1 is the bending stiffness of plate 1, k_1 is the wave number of plate 1 for an angular frequency of ω, η_1 is the in situ damping loss factor of plate 1 and $k_i = k_a \sin\phi$ is the wave number of the plane wave forced by an airborne plane wave of wave number k_a which is incident on plate 1 at an angle of ϕ to the normal to plate 1. The damping loss term has been added to the Villot and Guigou-Carter (2000) equation.

If the rms amplitude of the incident airborne diffuse sound field at the plate surface is 1, then from equation (11) of Villot and Guigou-Carter (2000),

$$\begin{aligned}|v_{1+}|^2 &= \frac{\omega^2}{B_1^2\left\{\left(k_i^4-k_1^4\right)^2+\eta_1{}^2k_i^8\right\}} \\ &= \frac{1}{\omega^2 m_1^2\left\{\left(\chi_a^4-1\right)^2+\eta_1{}^2\chi_a^8\right\}}\end{aligned} \tag{4.169}$$

Thus

$$\frac{IB_1^2}{2m_2c_2\omega^2} = \frac{|t|^2\left(\sqrt{1-\dfrac{s^2}{\kappa^2}}\right)}{\left(k_a^4\sin^4\phi-k_1^4\right)^2+\eta^2k_a^8\sin^8\phi}, \tag{4.170}$$

and

$$\begin{aligned}s^2 &= \chi_a^2\sin^2\theta_i \\ &= \frac{k_a^2\sin^2\phi\sin^2\theta_i}{k_1^2}.\end{aligned} \tag{4.171}$$

For a diffuse field acoustic incident wave equation (4.171) has to be averaged over ϕ and θ_i.

$$Av\left(\frac{IB_1^2}{2m_2c_2\omega^2}\right) = \frac{2}{\pi}\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\frac{|t|^2\,\mathrm{Re}\left(\sqrt{1-\dfrac{s^2}{\kappa^2}}\right)\sin\phi}{\left(k_a^4\sin^4\phi-k_1^4\right)+\eta^2k_a^8\sin^8\phi}\;d\theta_i d\phi. \tag{4.172}$$

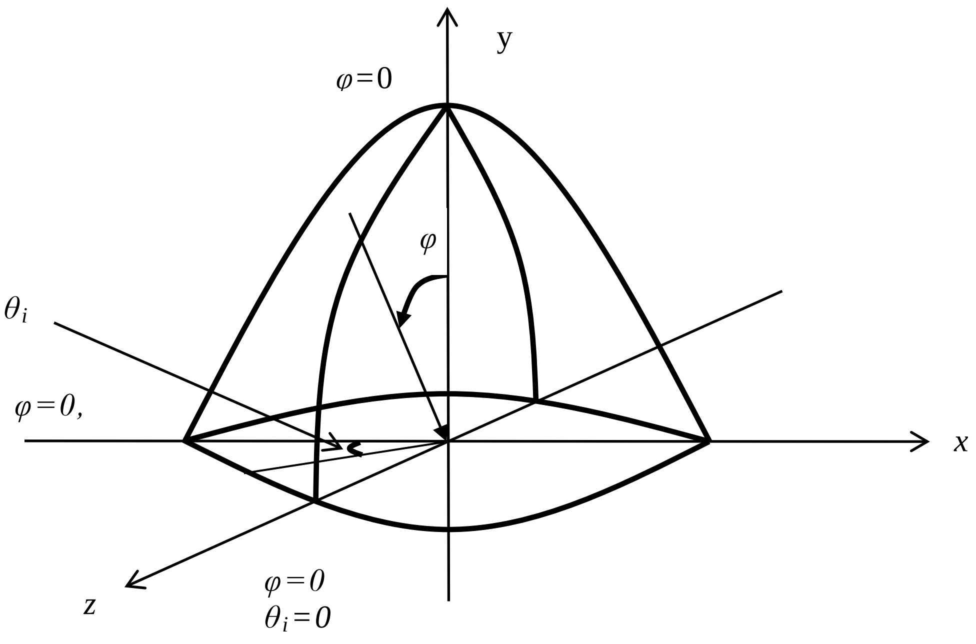

Figure 4.4 The octant that equation 4.187 is averaged over.

dblint.m calculates the values of the integral that is proportional to the power per unit length transmitted through the junction when plate 1 is excited by a diffuse acoustic sound field (see appendix 1). The integral is conducted only from 0 to π/2 which is the solid angle of the octant over which the integration is made. Note that the angular weighting is different from that used in equation (13) by Villot and Guigou-Carter (2000). In the notation of this thesis, Villot and Guigou-Carter evaluate an integral of the form.

$$\int_0^{k_a} \int_0^{\frac{\pi}{2}} F(\theta, k_i) k_i d\theta_i dk_i. \tag{4.173}$$

Since

$$k_i = k_a \sin\phi, \quad dk_i = k_a \cos\phi. \tag{4.174}$$

Thus their integral becomes

$$\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} F(\theta, k_a \sin\phi) k_a^2 \sin\phi \cos\phi d\theta_i d\phi \tag{4.175}$$

The k_a squared and the $2/\pi$ are just constants which can be corrected for but the $\cos\varphi$ is incorrect because the forced bending wave velocity depends on the incident acoustic power.

4.10 Summary

The reflected wave amplitudes (r), the reflected nearfield wave amplitudes (r_j), the transmotted wave amplitudes (t), and the transmitted nearfield (t_j) waves are derived. These wave amplitudes are calculated for the situation of normal incidence. Then the wave number equations from *Cremer et al*. (2005) are derived for the obliquely incident freely propagating wave case. Next, the angular velocity and the torsional moment in the x direction are determined by values in the y and z directions. This is shown for the obliquely incident wave case. Then the derivation of the transmitted and reflected wave amplitude for the obliquely incident forced wave is given. These amplitudes are used to calculate the transmitted bending wave intensity. Then the intensity transmitted by a diffuse bending wave field is calculated. Finally, the transmitted intensity when plate 1 is excited by a diffuse acoustic field is derived. This final calculation was the main aim of this thesis.

Chapter 5 The results for pinned plates.

In this chapter, graphs of the relative transmitted intensity at the junction of two semi-finite in the plane plates will be presented. As explained in chapters 2 and 3, and in section 4.6 it is not possible to calculate meaningful intensities for the incident and reflected waves unless the incident wave is freely propagating. Only the total intensity, which is incident on the junction can be meaningfully calculated and this is equal to the transmitted intensity. Thus only the transmitted intensity is considered in this chapter.

As in the previous chapter the plates will be assumed to have bending stiffnesses B_i and freely propagating wave numbers k_i where i=1 or 2. The ratio of the freely propagating wave numbers is

$$\kappa = \frac{k_2}{k_1}. \tag{5.1}$$

The variable ψ is defined to be

$$\psi = \frac{B_2 k_2^2}{B_1 k_1^2} = \frac{B_2}{B_1}\kappa^2. \tag{5.2}$$

The intensity of a freely propagating plane bending wave with transverse velocity v_i in the ith plate is (see Cremer *et al*. (2005) and section 4.7)

$$I_i = 2m_i c_i |v_i|^2, \tag{5.3}$$

where m_i is the mass per unit area and c_i is the freely propagating bending wave velocity of the ith plate. Thus

$$Z_i = 2m_i c_i \tag{5.4}$$

can be interpreted as the impedance experienced by a freely propagating bending wave. For a freely propagating bending wave Cremer *et al*. (2005)

$$\frac{B_i}{m_i} k_i^4 = \omega^2, \tag{5.5}$$

and

$$\frac{\omega}{k_i} = c_i, \tag{5.6}$$

where ω is the angular frequency.

Equations (5.4), (5.5) and (5.6) and give

$$B_i k_i^2 = m_i c_i^2 = \frac{Z_i c_i}{2}. \tag{5.7}$$

Hence ψ can be written as

$$\psi = \frac{Z_2 c_2}{Z_1 c_1} = \frac{Z_2}{Z_1} \frac{k_1}{k_2} = \frac{Z_2}{Z_1} \frac{1}{\kappa} \tag{5.8}$$

Thus ψ can be interpreted as the ratio of the "freely propagating bending wave impedances" divided the ratio of the freely propagating wave numbers.

The forced incident wave will be in plate 1 and have a forced wave number of k_i. It will be incident at an angle of θ_i to the normal to the line junction between the two semi-finite in plane plates. The variable χ_a is the ratio of the forced bending wave number of the incident wave to the freely propagating wave number in plate 1.

$$\chi_a = \frac{k_i}{k_1}. \tag{5.9}$$

The freely propagating wave case is given by χ_a equals 1. First graphs of the relative transmitted intensity (relative power per unit length of the junction) will be given. Equation (4.160) will be graphed.

This equation is

$$\frac{I}{2\left|v_{1+}\right|^2 m_2 c_2} = \left|t\right|^2 \operatorname{Re}\left(\sqrt{1-\left(\frac{\chi_a}{\kappa}\right)^2 \sin^2\theta_i}\right). \tag{5.10}$$

Thus it can be seen that the transmitted intensity is either normalized by or is relative to the intensity of a freely propagating plane bending wave. This reference wave has the same transverse bending wave velocity as the incident wave but is propagating in plate 2 at a transmitted angle of 0^o relative to the normal of the line junction.

The transmitted intensity has been further normalized by dividing it by the transmitted intensity for the freely propagating incident wave case (χ_a=1). This is not possible in all these graphs because sometimes the transmitted intensity for the freely propagating incident wave case is zero. In these cases the transmitted intensity is normalized by dividing it by its maximum value. This further normalization makes the curves in each figure nearly over lay each other.

It can be seen immediately that the relative transmitted intensity is zero when

$$\chi_a \geq \frac{\kappa}{\left|\sin\theta_i\right|}. \tag{5.11}$$

In this case total internal reflection occurs. The minimum value of χ_a for which total internal reflection occurs as a function of κ and $\left|\theta_i\right|$ are given in the following table 5.1.

	κ=1/2	κ=1	κ=2
$\lvert\theta_i\rvert = 0^o$	∞		
$\lvert\theta_i\rvert = 15^o$	1.93	3.86	7.73
$\lvert\theta_i\rvert = 30^o$	1	2	4
$\lvert\theta_i\rvert = 45^o$	0.707	1.41	2.83
$\lvert\theta_i\rvert = 60^o$	0.577	1.15	2.31
$\lvert\theta_i\rvert = 75^o$	0.518	1.04	2.07
$\lvert\theta_i\rvert = 90^o$	0.5	1	2

Table 5.1 The minimum value of χ_a for which total internal reflection occurs for different values of κ and θ_i.

The root mean square complex amplitude of the freely propagating transmitted wave in plate 2 divided by the root mean square complex amplitude of the forced incident wave in plate 1 is t. From equation (4.130)

$$t = \frac{\left(j\chi_a \cos\theta_i + \sqrt{1+\chi_a^2 \sin^2\theta_i}\right) + \begin{pmatrix} j\sqrt{1-\chi_a^2 \sin^2\theta_i} \\ -\sqrt{1+\chi_a^2 \sin^2\theta_i} \end{pmatrix} \left(\chi_a^2 + 1\right)/\,2}{\begin{array}{c} \psi\left(j\sqrt{1-\chi_a^2 \sin^2\theta_i} - \sqrt{1+\chi_a^2 \sin^2\theta_i}\right) \\ + \left(j\sqrt{\kappa^2 - \chi_a^2 \sin^2\theta_i} - \sqrt{\kappa^2 + \chi_a^2 \sin^2\theta_i}\right) \end{array}}. \tag{5.12}$$

The only one of the terms in the numerator of the right hand side of equation (5.12) that can possibly become zero is

$$j\sqrt{1-\chi_a^2 \sin\theta_i}. \tag{5.13}$$

Equation (5.13) is zero when

$$\chi_a = \frac{1}{|\sin\theta_i|}. \tag{5.14}$$

It can be seen from equations (4.62) and (4.63), that the value of χ_a given by equation (5.14) is the maximum value of χ_a for which the reflected propagating wave is actually a wave. For values of χ_a greater than that given by equation (5.14) the propagating reflected wave becomes a non-propagating nearfield wave.

At the value of χ_a given by equation (5.14) it is likely that the graphs will exhibit a local minimum because the graphs are proportional to $|t|^2$. However this local minimum will not be observed if values of χ_a given by equation (5.14) are greater than or equal to the value of χ_a given by the equals sign in equation (5.11). This will happen when κ is equal to or less than one. This is because the real function in equation (5.10) makes the relative transmitted intensity zero in the region of the value of χ_a where a local minimum would probably have occurred. The values of χ_a for which the local minimum will probably occur are given in table 5.2.

	κ>1
$\lvert\theta_i\rvert = 0^o$	
$\lvert\theta_i\rvert = 15^o$	3.86
$\lvert\theta_i\rvert = 30^o$	2
$\lvert\theta_i\rvert = 45^o$	1.41
$\lvert\theta_i\rvert = 60^o$	1.15
$\lvert\theta_i\rvert = 75^o$	1.04
$\lvert\theta_i\rvert = 90^o$	1

Table 5.2 Values of χ_a for which a local minimum in the transmitted intensity will probably occur for different values of θ_i.

Apart from the moduli of the term given by equation (5.13), all the moduli of the terms in the numerator of equation (5.12) increase as χ_a increases. Thus apart from the possible local minimum and the ultimate driving to zero (except for θ_i=0) by the real function in equation (5.10), the graphs are expected to be increasing functions of χ_a. Figures 5.1 to 5.3 show the relative transmitted intensity at 0˚ for κ equals ½, 1 and 2 respectively. Each next set of three figures shows the same graphs for the case when θ_i has been increased by 15˚. This trend is continued until θ_i equals 90˚.

Figures 5.1 to 5.21 show a wide variety of behaviour. Except for the normal incidence case, total internal reflection occurs for smaller values of χ_a as κ decreases and θ increases. On most of the graphs the curves for different values of ψ are almost identical. These curves are only significantly different on the κ equals 2 graphs, for the larger values of χ_a and θ. The volume of χ_a for which a minimum occurs in some of the graphs is well predicted by values given in table 5.2.

The relative bending intensity (power per unit length of the junction) transmitted by a forced diffuse field in plate 1 will now be calculated. This is done by integrating equation (5.10) over angles of incidence θ_i ranging from 0 radians up to the angle at which total internal reflection occurs or π/2 radians (90˚) if total internal reflection does not occur. Thus the upper limit of integration θ_u is given by equation (4.166)

$$\theta_u = \begin{cases} \dfrac{\pi}{2} & \text{if } \kappa \geq \chi_a \\ \arcsin\left(\dfrac{\kappa}{\chi_a}\right) & \text{if } \kappa < \chi_a \end{cases}. \tag{5.15}$$

Hence the relative intensity is proportional to equation (4.165)

$$\int_0^{\theta_u} |t|^2 \operatorname{Re}\left[\sqrt{1-\left(\frac{\chi_a}{\kappa}\right)^2 \sin^2\theta_i}\right] d\theta_i. \tag{5.16}$$

Equation (5.16) will always give non-zero results. Thus the result of equation (5.16) will be further normalized by dividing it by the result for a freely propagating incident wave (χ_a=1). These results will then be converted to decibels and graphed in figures 5.22 to 5.24. The normalized curves in these figures overlay each other except when χ_a is greater than one in the κ equals two case, shown in figure 5.24. This outcome can be predicted from the results shown in figures 5.1 to 5.21.

Except for the local minimum in the κ=1/2 results shown in figure 5.22, the curves are increasing functions of χ_a. The local minimum in the κ=1/2 case appears to be an interaction between the increasing nature of the relative transmission as a function of χ_a for angles of incidence close to normal incidence and the decrease in the angle at which total internal reflection occurs as a function of χ_a.

Next it is assumed that the bending wave field in plate 1 is forced by an incidence diffuse acoustic field. The forced bending wave number k_i will vary from k_a, where k_a is the value of the wave number of the diffuse sound field. The variable r is defined to be the ratio of the airborne wave number k_a to the freely propagating wave number k_1 in plate 1.

$$r = \frac{k_a}{k_1}. \tag{5.17}$$

In this case the relative transmitted bending wave intensity is proportional to equation (4.172).

$$\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \frac{|t|^2 \operatorname{Re}\left(\sqrt{1-\left(\frac{\chi_a}{\kappa}\right)^2 \sin^2\theta_i}\right)\sin\phi}{\left(k_a^4\sin^4\phi - k_1^4\right)+\eta^2 k_a^8 \sin^8\phi}\, d\theta d\phi. \tag{5.18}$$

The $2/\pi$ from equation (4.172) has been dropped because equation (5.24) will be normalized by dividing it by its value for the freely propagating incidence wave case when χ_a equals one. Note that the angular weighting is different from that used in equation (13) of by Villot and Guigou-Carter (2000).

After normalization, the value of equation (5.18) will be converted to decibels before being graphed in figures 5.25 to 5.27 where the curves in each figure overlay each other. This is because the curves in figures 5.1 to 5.21 overlay each other, apart from the κ=2 case when the values of χ_a are greater than one. Figures 5.25 to 5.27 differ from figures 5.22 to 5.24 because a weighted average of the transmitted intensity has been taken over by values of χ_a from zero to r.

For values of r greater than one this average is dominated by values of χ_a close to one. In this case:

$$k_a|\sin\phi| \approx k_1. \tag{5.19}$$

Because the damping loss factor η is small (0.03), the denominator of the integrand in equation (5.18) is close to zero when equation (5.24) applies. This means that the integrad has a sharp maximum at χ_a=1. The effect is that the integral is relatively constant for r greater than or equal to one. Because the integral is normalized by dividing by its value when r equals one, the normalized integral is close to one which is 0 dB when r is greater than one.

When r is less than one, the curves follow the general behavior of figures 5.22 to 5.24. That is, they are increasing functions of r except for a local minimum in the $\kappa=1/2$ case.

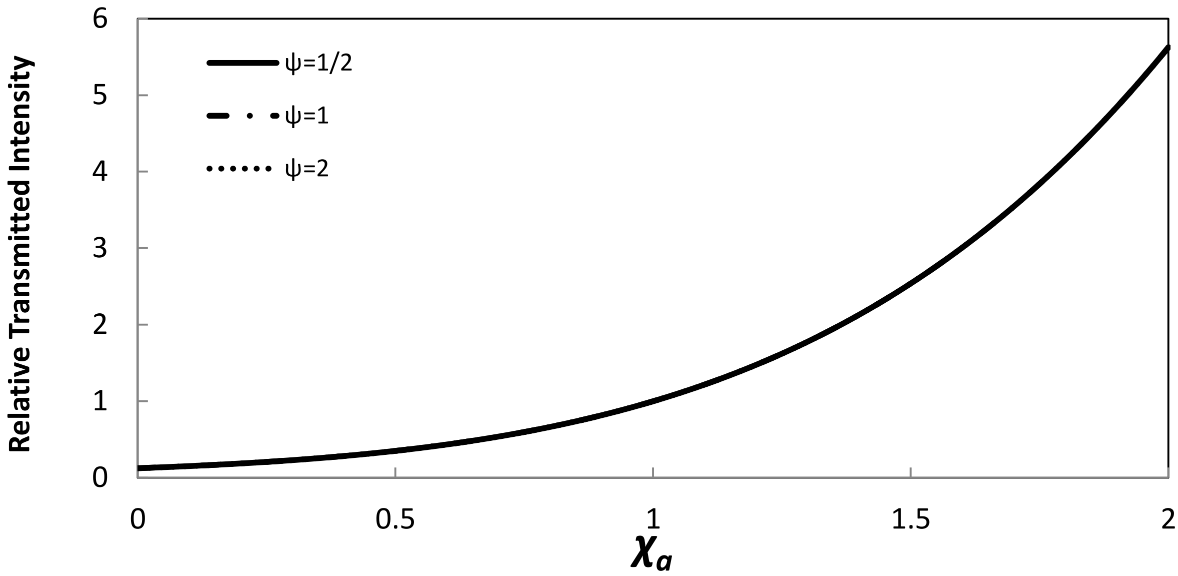

Figure 5.1 The relative transmitted intensity at the junction of two infinite panels due to a forced wave in the first panel incident at an angle of incidence to the normal of 0˚. The ratio κ of the wave number in the second panel to that in the first panel is ½. Curves are given for the ratio ψ equals ½, 1 and 2.

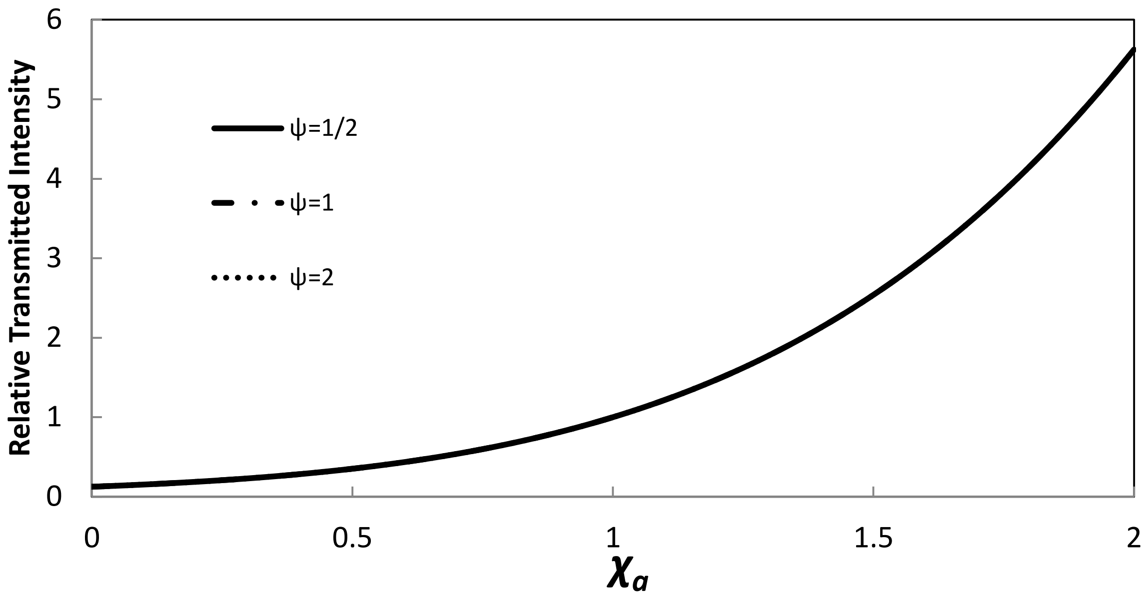

Figure 5.2 . The relative transmitted intensity at the junction of two infinite panels due to a forced wave in the first panel incident at an angle of incidence to the normal of 0°. The ratio κ of the wave number in the second panel to that in the first panel is 1. Curves are given for the ratio ψ equals ½, 1 and 2.

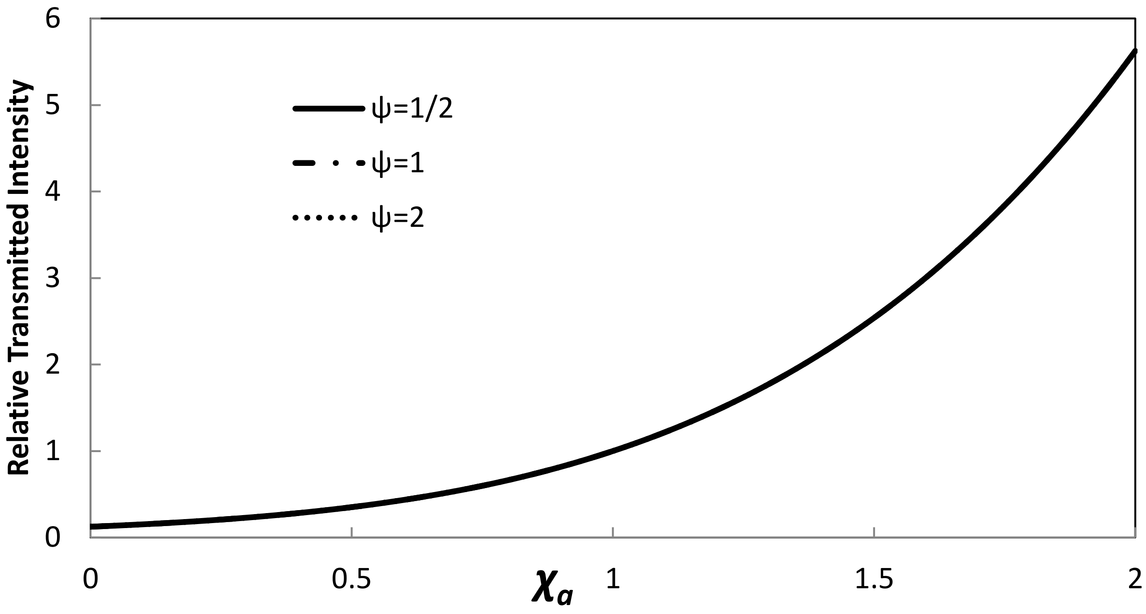

Figure 5.3 The relative transmitted intensity at the junction of two infinite panels due to a forced wave in the first panel incident at an angle of incidence to the normal of 0°. The ratio κ of the wave number in the second panel to that in the first panel is 2. Curves are given for the ratio ψ equals ½, 1 and 2.

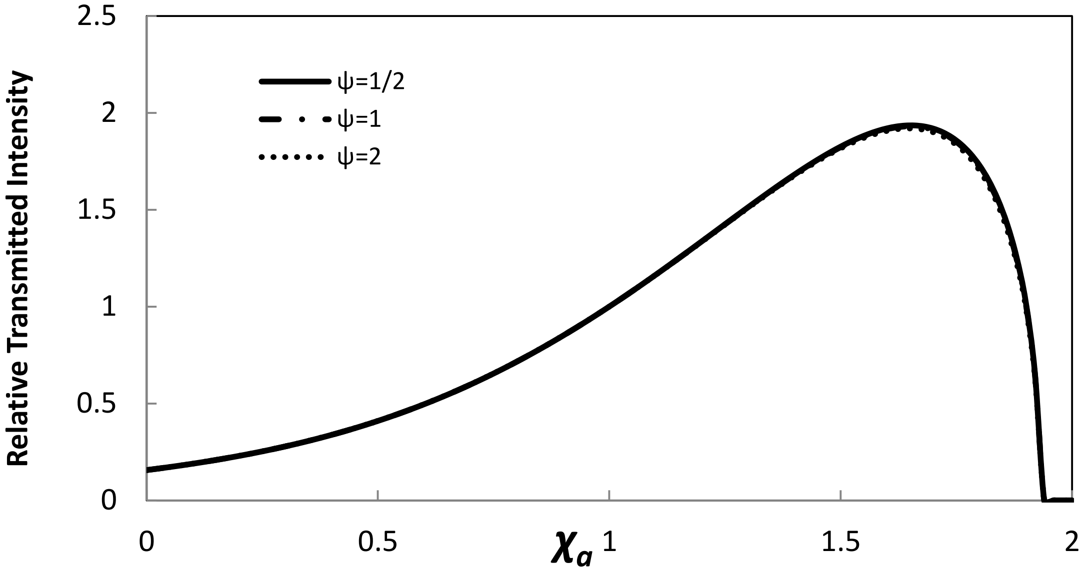

Figure 5.4 The relative transmitted intensity at the junction of two infinite panels due to a forced wave in the first panel incident at an angle of incidence to the normal of 15˚. The ratio κ of the wave number in the second panel to that in the first panel is 1/2. Curves are given for the ratio ψ equals ½, 1 and 2.

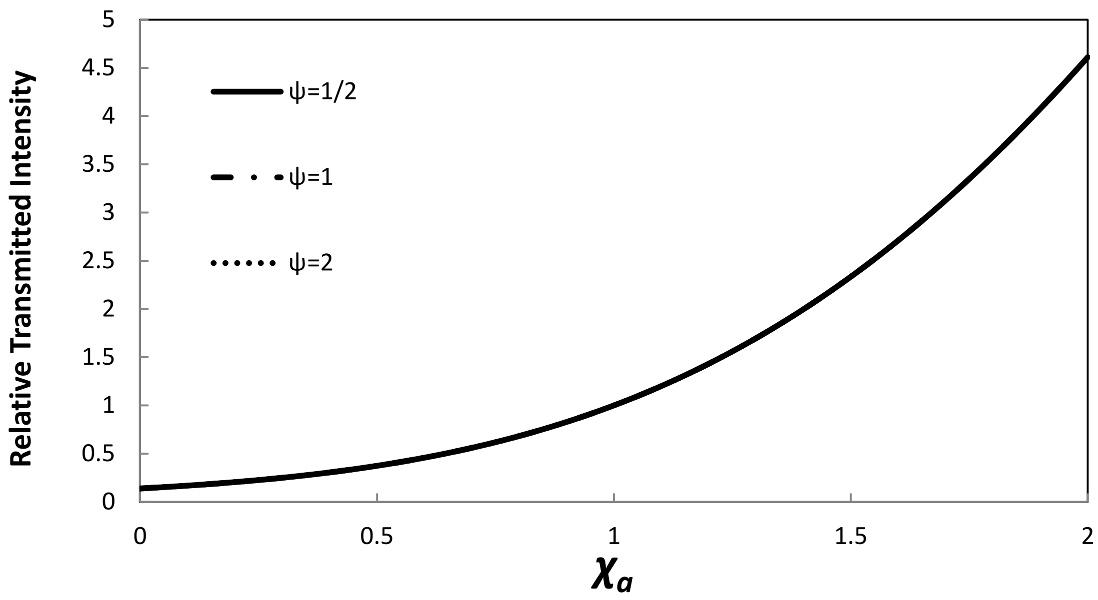

Figure 5.5 The relative transmitted intensity at the junction of two infinite panels due to a forced wave in the first panel incident at an angle of incidence to the normal of 15˚.The ratio κ of the wave number in the second panel to that in the first panel is 1. Curves are given for the ratio ψ equals ½, 1 and 2.

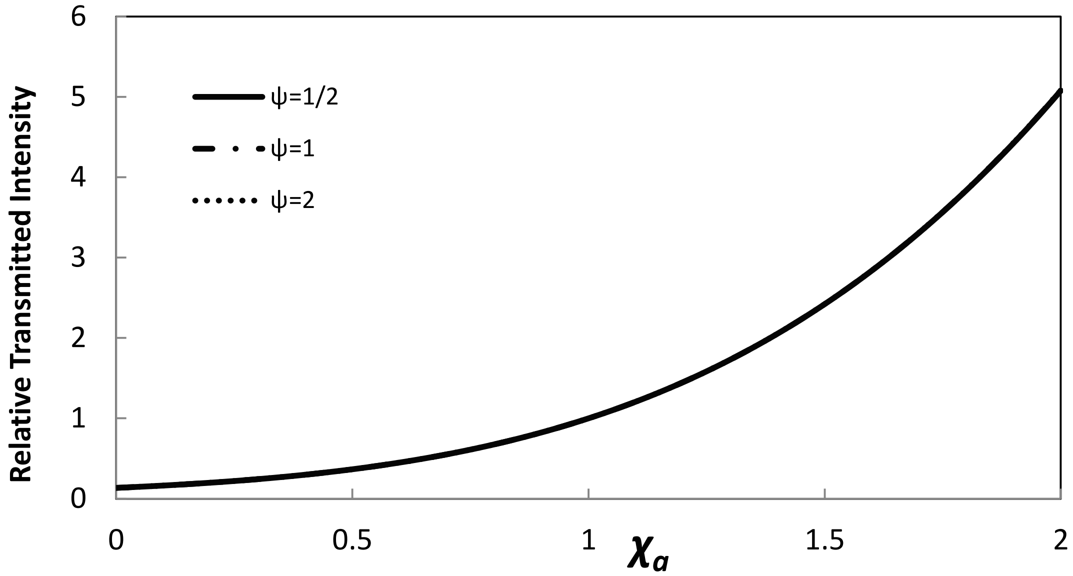

Figure 5.6 The relative transmitted intensity at the junction of two infinite panels due to a forced wave in the first panel incident at an angle of incidence to the normal of 15˚. The ratio κ of the wave number in the second panel to that in the first panel is 2. Curves are given for the ratio ψ equals ½, 1 and 2.

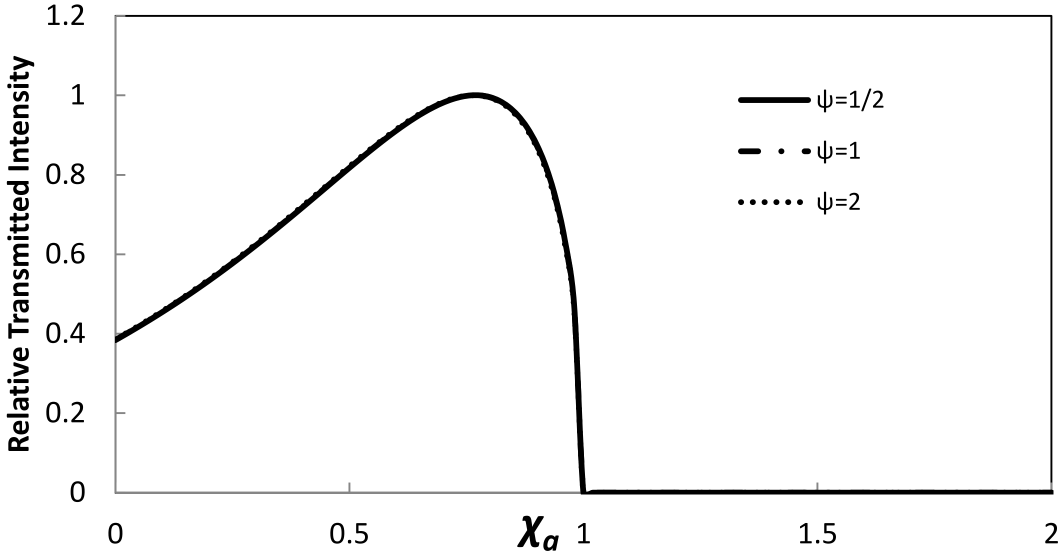

Figure 5.7 The relative transmitted intensity at the junction of two infinite panels due to a forced wave in the first panel incident at an angle of incidence to the normal of 30°. The ratio κ of the wave number in the second panel to that in the first panel is 1/2. Curves are given for the ratio ψ equals ½, 1 and 2.

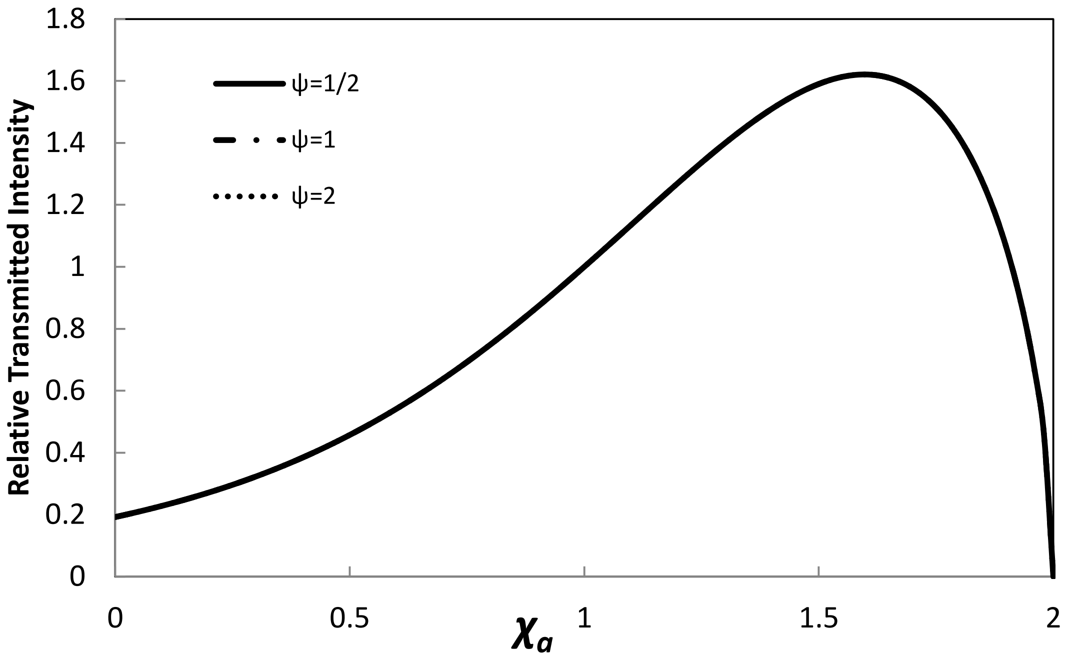

Figure 5.8 The relative transmitted intensity at the junction of two infinite panels due to a forced wave in the first panel incident at an angle of incidence to the normal of 30°. The ratio κ of the wave number in the second panel to that in the first panel is 1. Curves are given for the ratio ψ equals ½, 1 and 2.

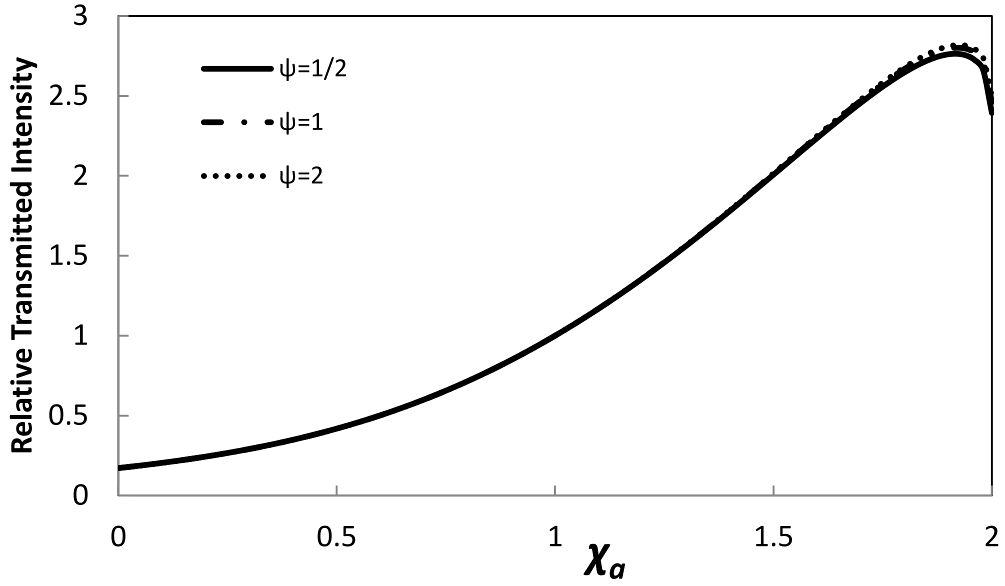

Figure 5.9 The relative transmitted intensity at the junction of two infinite panels due to a forced wave in the first panel incident at an angle of incidence to the normal of 30˚. The ratio κ of the wave number in the second panel to that in the first panel is 2. Curves are given for the ratio ψ equals ½, 1 and 2.

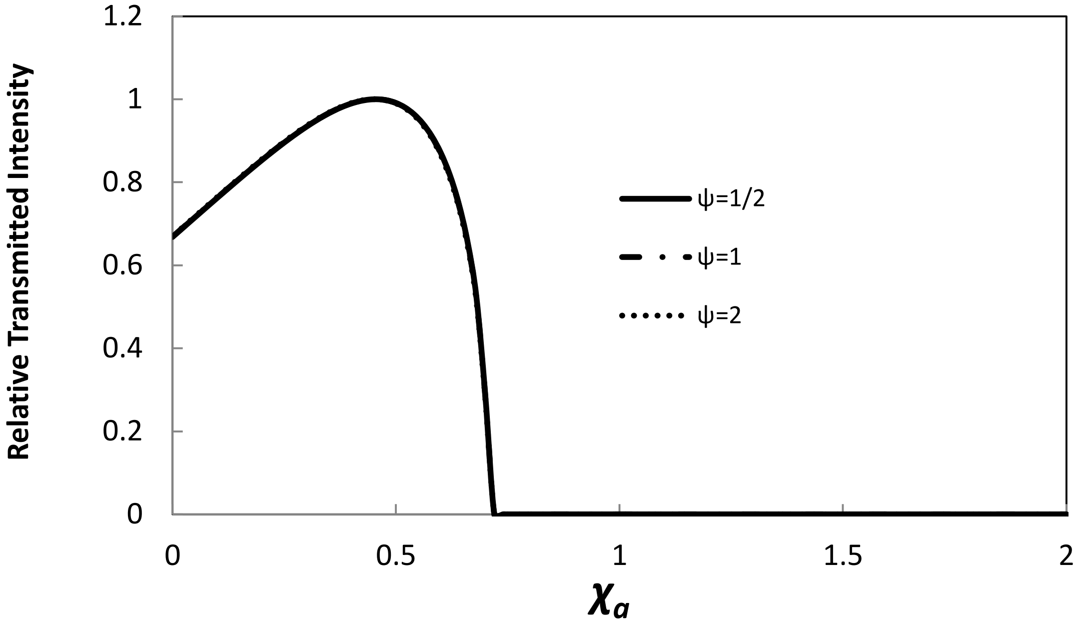

Figure 5.10 The relative transmitted intensity at the junction of two infinite panels due to a forced wave in the first panel incident at an angle of incidence to the normal of 45˚. The ratio κ of the wave number in the second panel to that in the first panel is 1/2. Curves are given for the ratio ψ equals ½, 1 and 2.

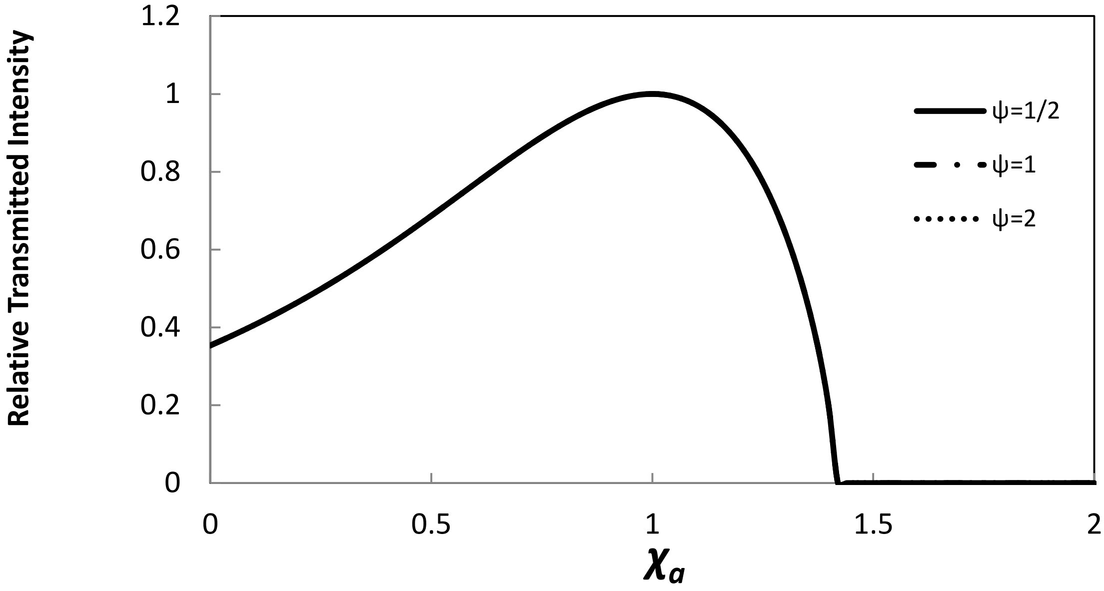

Figure 5.11 . The relative transmitted intensity at the junction of two infinite panels due to a forced wave in the first panel incident at an angle of incidence to the normal of 45°. The ratio κ of the wave number in the second panel to that in the first panel is 1. Curves are given for the ratio ψ equals ½, 1 and 2.

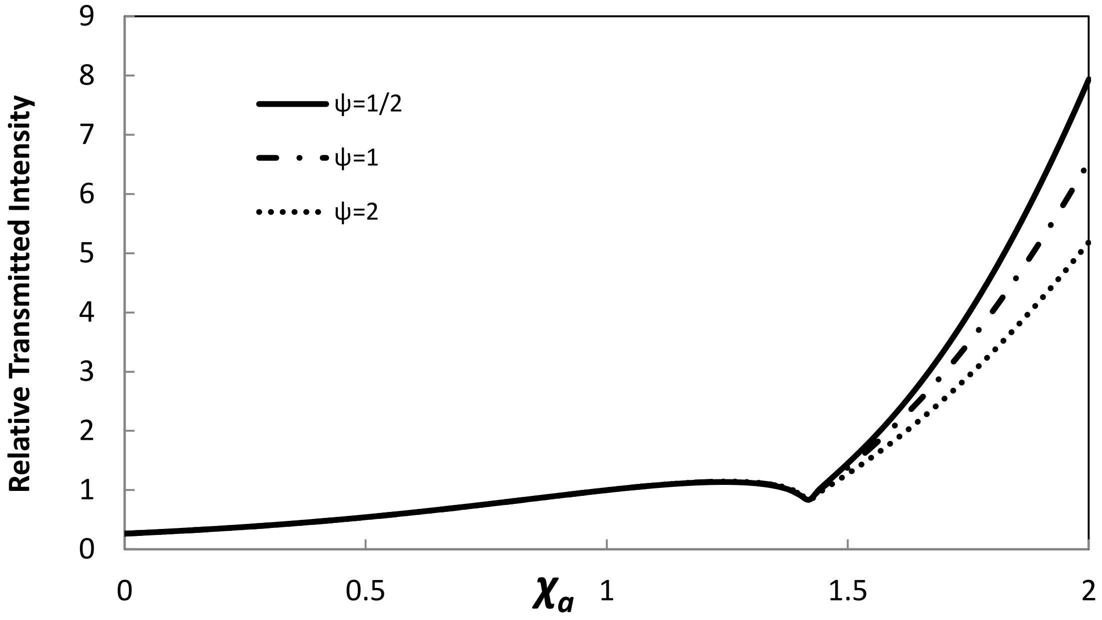

Figure 5.12 The relative transmitted intensity at the junction of two infinite panels due to a forced wave in the first panel incident at an angle of incidence to the normal of 45°. The ratio κ of the wave number in the second panel to that in the first panel is 2. Curves are given for the ratio ψ equals ½, 1 and 2.

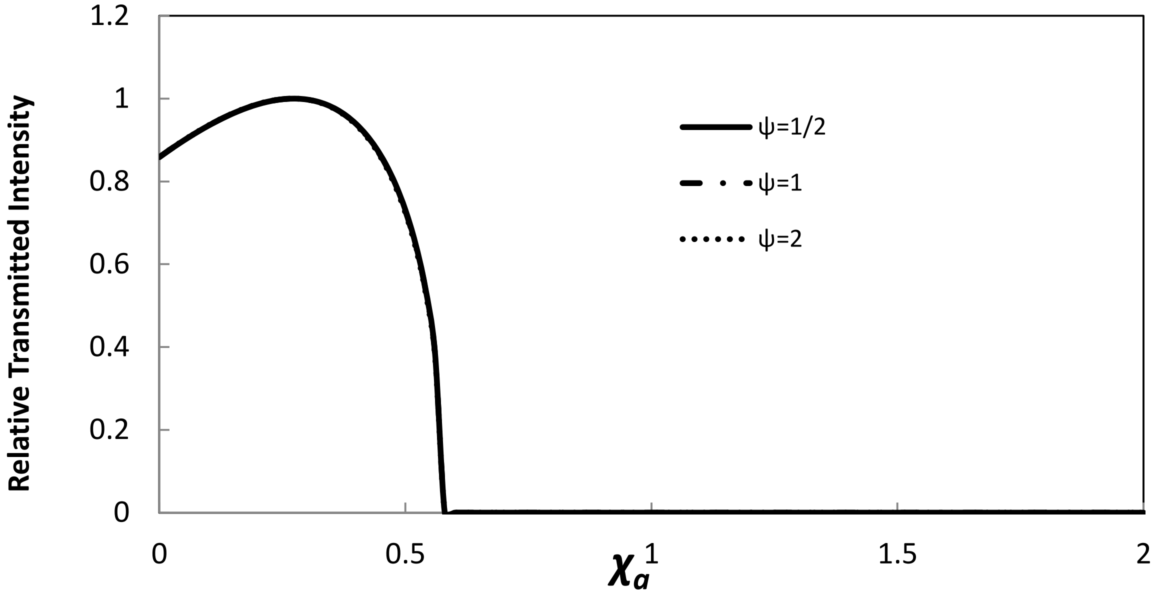

Figure 5.13 The relative transmitted intensity at the junction of two infinite panels due to a forced wave in the first panel incident at an angle of incidence to the normal of 60°. The ratio κ of the wave number in the second panel to that in the first panel is 1/2. Curves are given for the ratio ψ equals ½, 1 and 2.

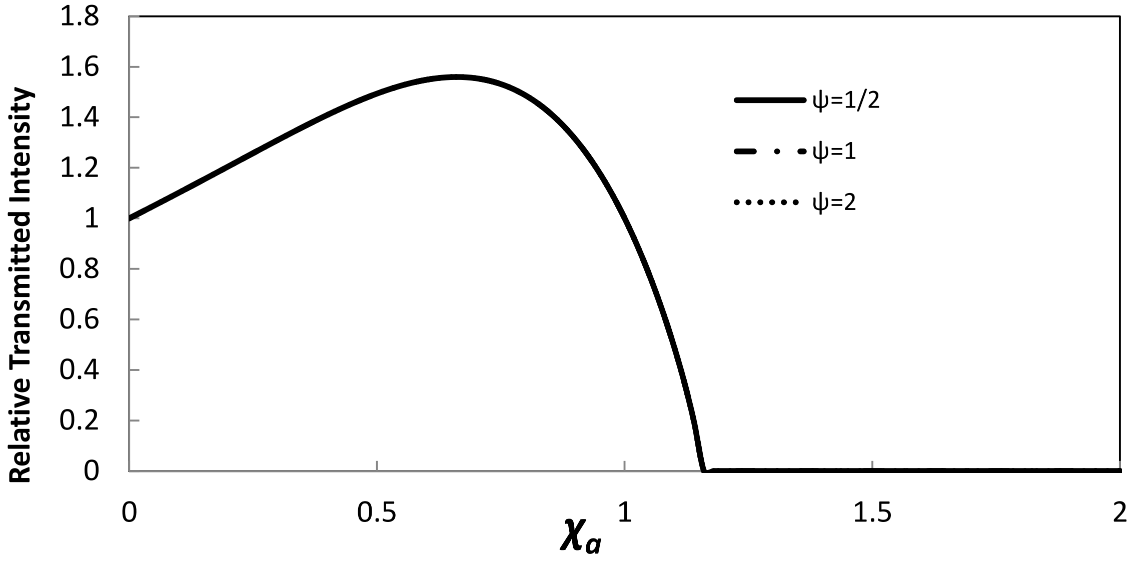

Figure 5.14 The relative transmitted intensity at the junction of two infinite panels due to a forced wave in the first panel incident at an angle of incidence to the normal of 60˚. The ratio κ of the wave number in the second panel to that in the first panel is 1. Curves are given for the ratio ψ equals ½, 1 and 2.

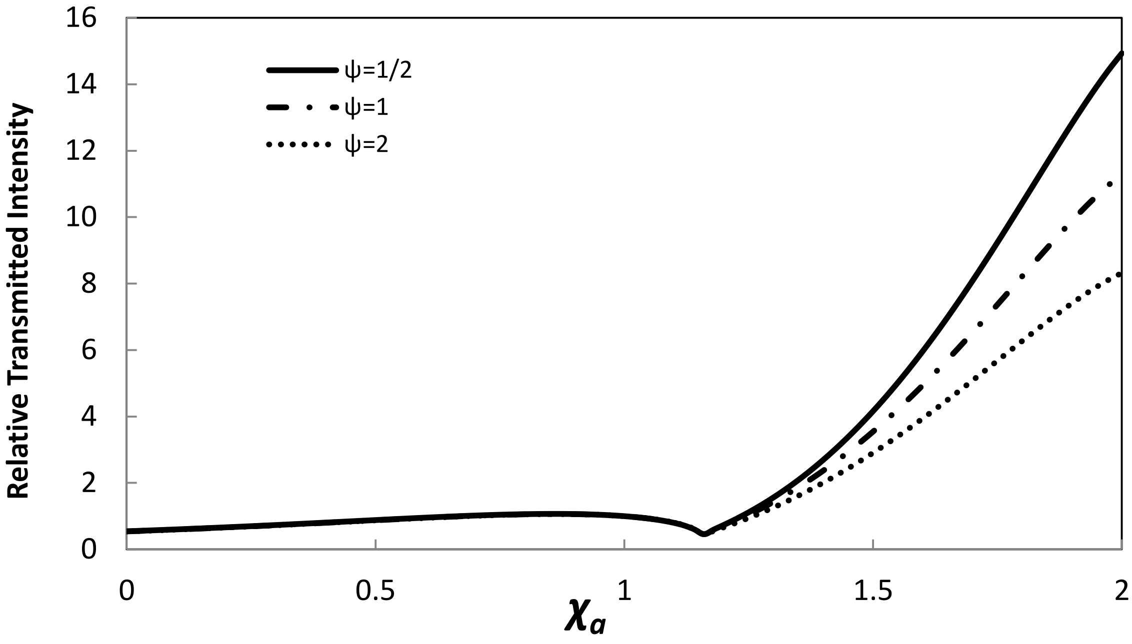

Figure 5.15 The relative transmitted intensity at the junction of two infinite panels due to a forced wave in the first panel incident at an angle of incidence to the normal of 60°. The ratio κ of the wave number in the second panel to that in the first panel is 2. Curves are given for the ratio ψ equals ½, 1 and 2.

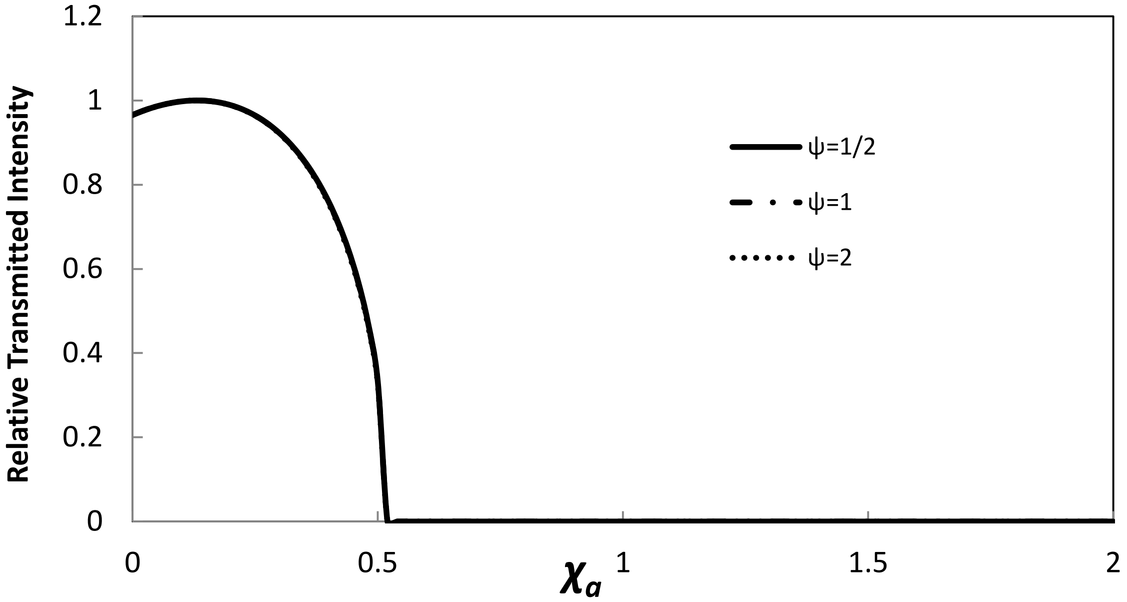

Figure 5.16 . The relative transmitted intensity at the junction of two infinite panels due to a forced wave in the first panel incident at an angle of incidence to the normal of 75˚. The ratio κ of the wave number in the second panel to that in the first panel is 1/2. Curves are given for the ratio ψ equals ½, 1 and 2.

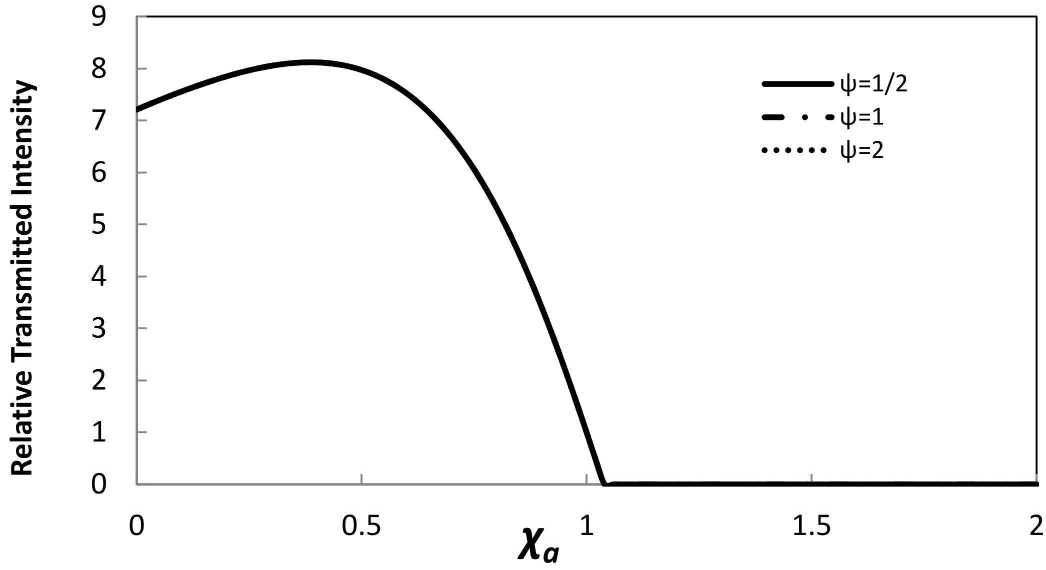

Figure 5.17 . The relative transmitted intensity at the junction of two infinite panels due to a forced wave in the first panel incident at an angle of incidence to the normal of 75˚. The ratio κ of the wave number in the second panel to that in the first panel is 1. Curves are given for the ratio ψ equals ½, 1 and 2.

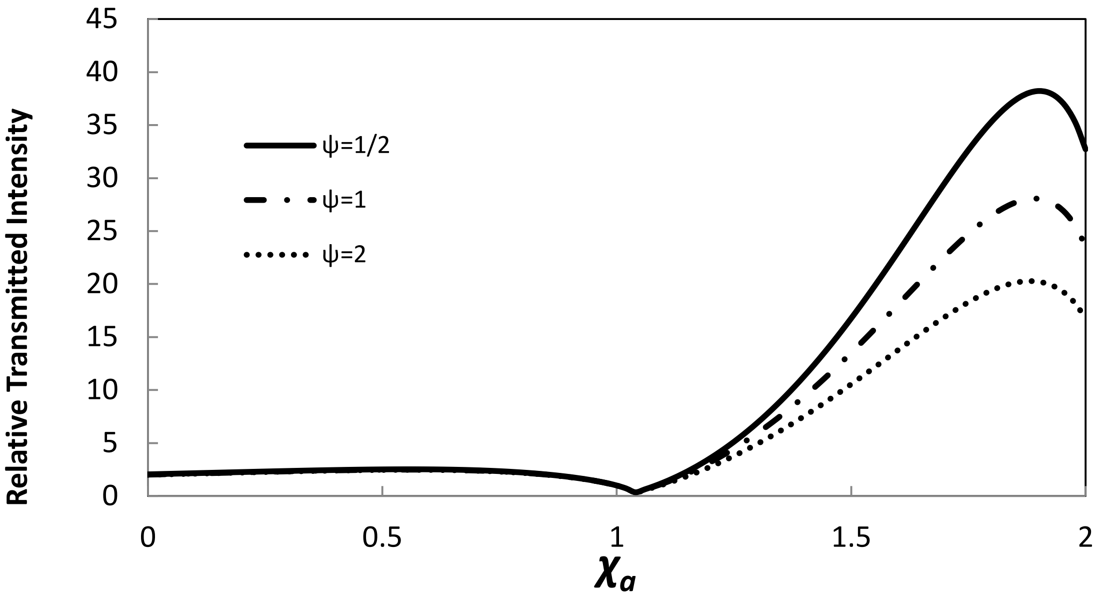

Figure 5.18 The relative transmitted intensity at the junction of two infinite panels due to a forced wave in the first panel incident at an angle of incidence to the normal of 75°. The ratio κ of the wave number in the second panel to that in the first panel is 2. Curves are given for the ratio ψ equals ½, 1 and 2.

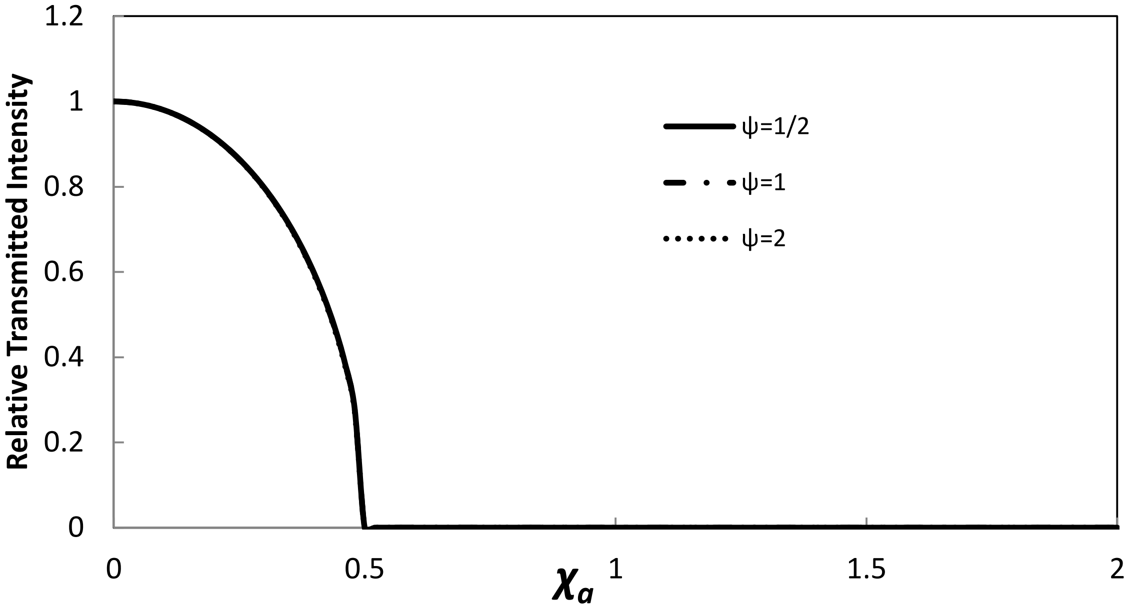

Figure 5.19 . The relative transmitted intensity at the junction of two infinite panels due to a forced wave in the first panel incident at an angle of incidence to the normal of 90˚. The ratio κ of the wave number in the second panel to that in the first panel is 1/2. Curves are given for the ratio ψ equals ½, 1 and 2.

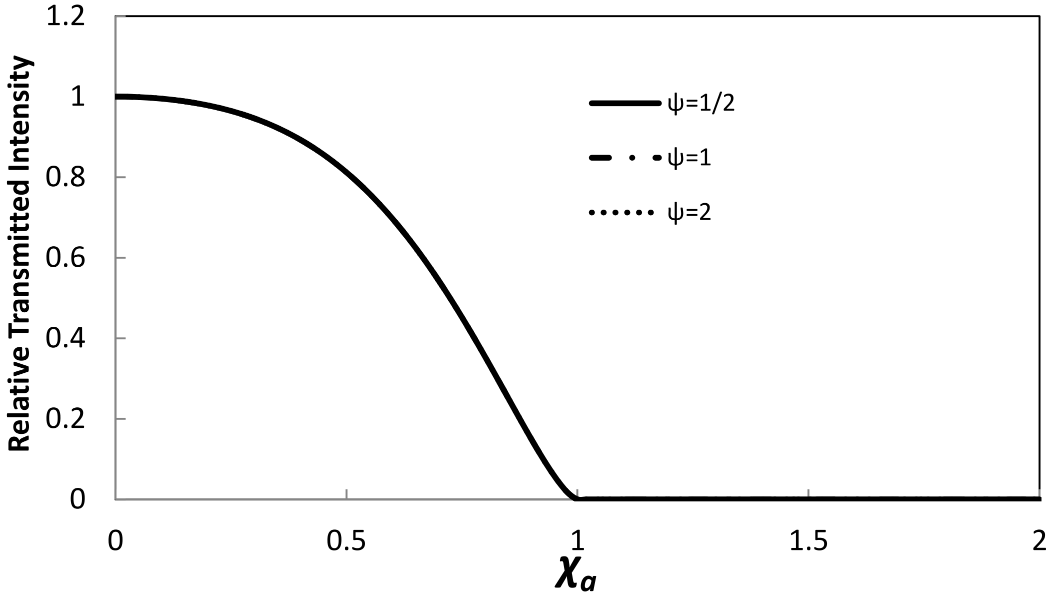

Figure 5.20 The relative transmitted intensity at the junction of two infinite panels due to a forced wave in the first panel incident at an angle of incidence to the normal of 90°. The ratio κ of the wave number in the second panel to that in the first panel is 1. Curves are given for the ratio ψ equals ½, 1 and 2.

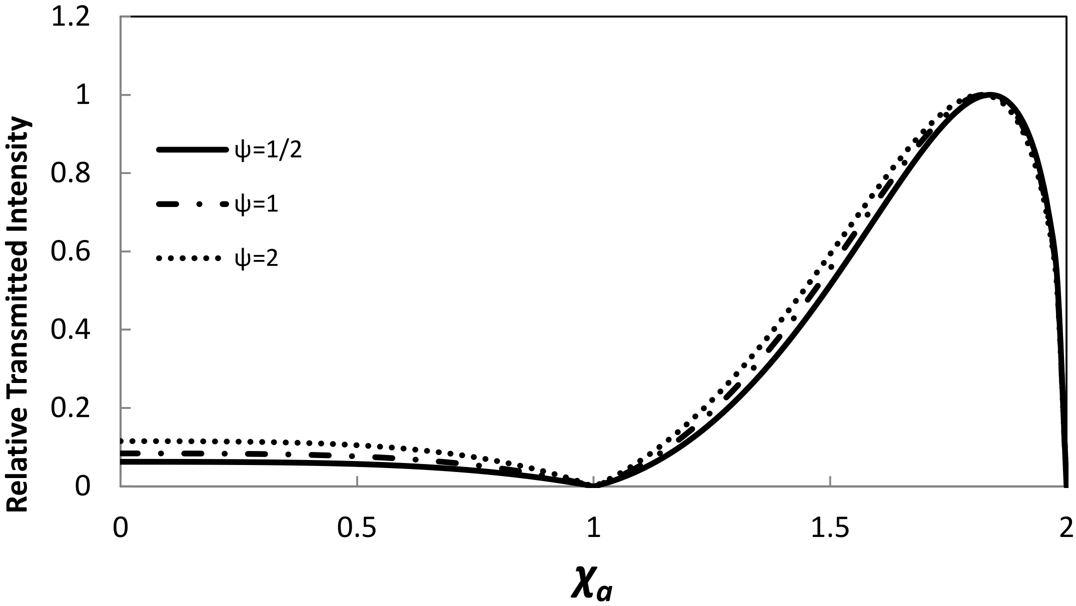

Figure 5.21 The relative transmitted intensity at the junction of two infinite panels due to a forced wave in the first panel incident at an angle of incidence to the normal of 90˚. The ratio κ of the wave number in the second panel to that in the first panel is 2. Curves are given for the ratio ψ equals ½, 1 and 2.

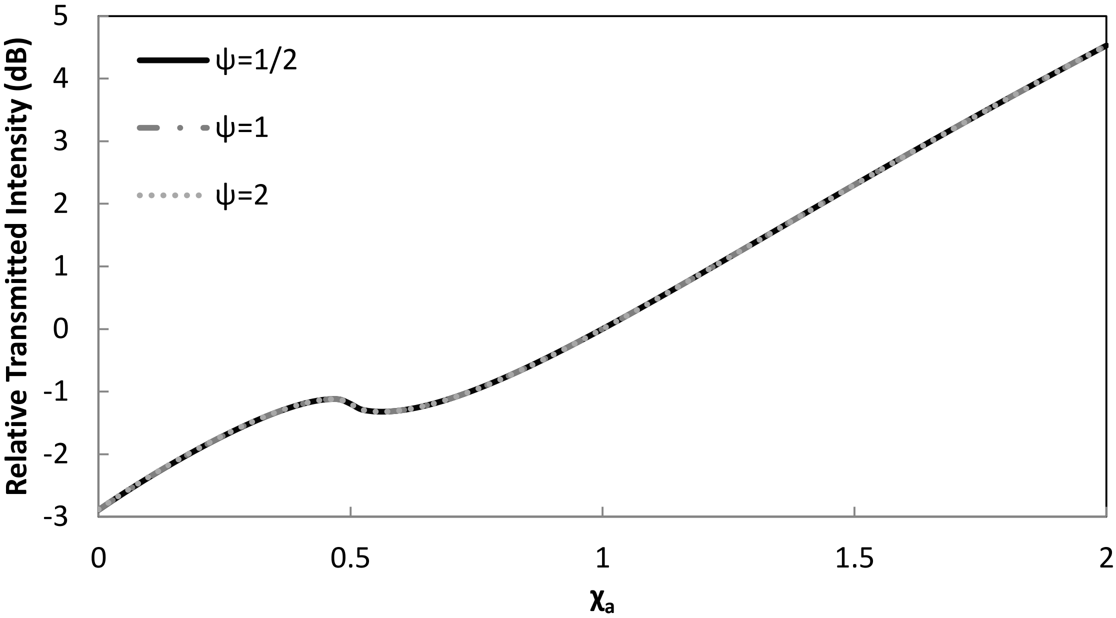

Figure 5.22 The incident field is a diffuse vibrational field. The integration is done over all the possible angles of incidence. Because of symmetry, the integration is only done from 0 to 90 degrees. The ratio κ of the wave number in the second panel to that in the first panel is 1/2. Curves are given for the ratio ψ equals ½, 1 and 2.

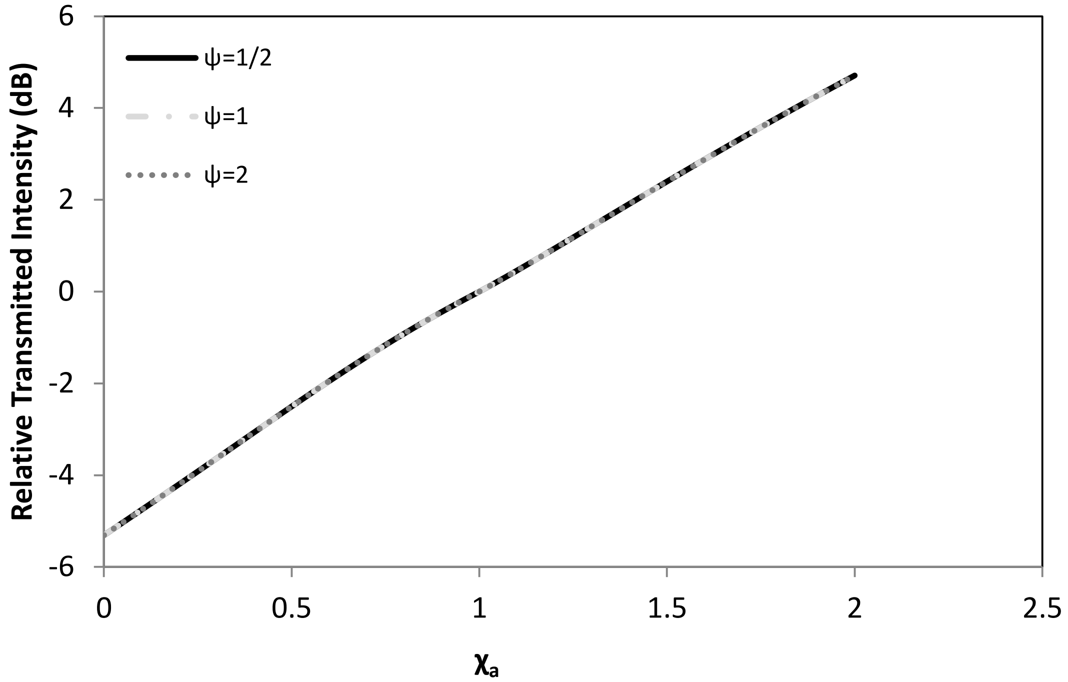

Figure 5.23 The incident field is a diffuse vibrational field. The integration is done over all the possible angles of incidence. Because of symmetry, the integration is only done from 0 to 90 degrees. The ratio κ of the wave number in the second panel to that in the first panel is 1. Curves are given for the ratio ψ equals ½, 1 and 2.

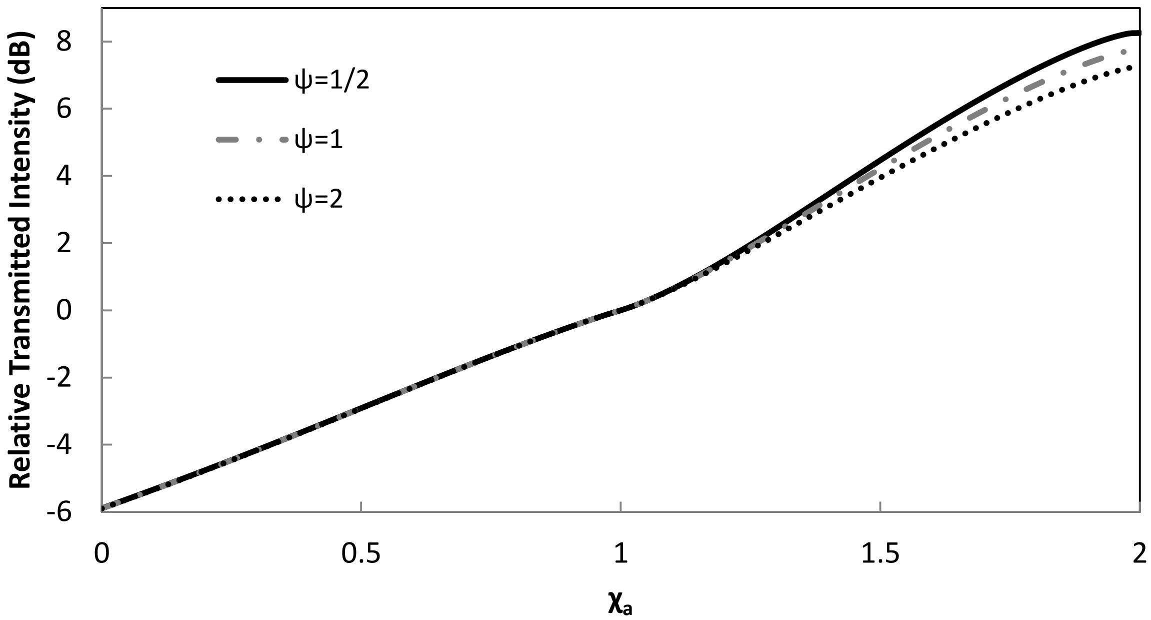

Figure 5.24 The incident field is a diffuse vibrational field. The integration is done over all the possible angles of incidence. Because of symmetry, the integration is only done from 0 to 90 degrees. The ratio κ of the wave number in the second panel to that in the first panel is 2. Curves are given for the ratio ψ equals ½, 1 and 2.

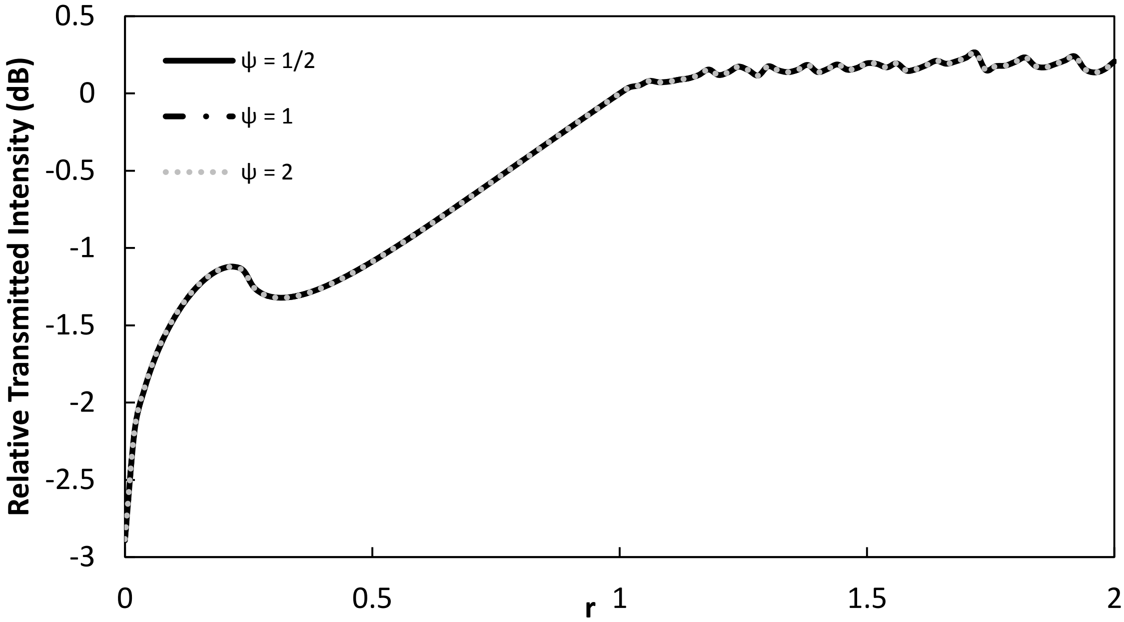

Figure 5.25 . The vibrational field in the 1st panel is excited by a diffuse incident acoustic field. The ratio κ of the wave number in the second panel to that in the first panel is 1/2. Curves are given for the ratio ψ equals ½, 1 and 2.

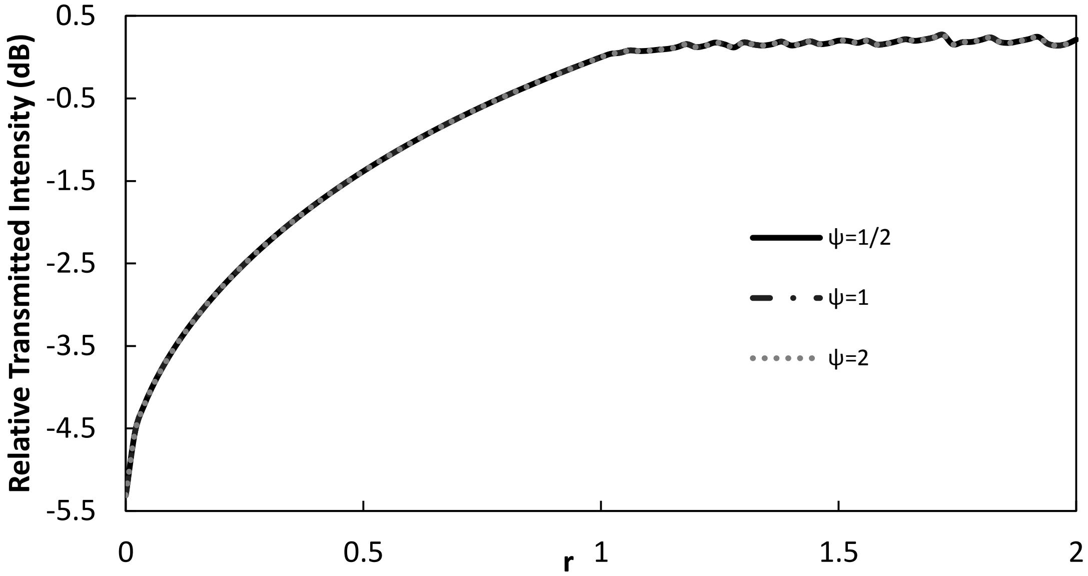

Figure 5.26 . The vibrational field in the 1st panel is excited by a diffuse incident acoustic field. The ratio κ of the wave number in the first panel to that in the second panel is 1. Curves are given for the ratio ψ equals ½, 1 and 2.

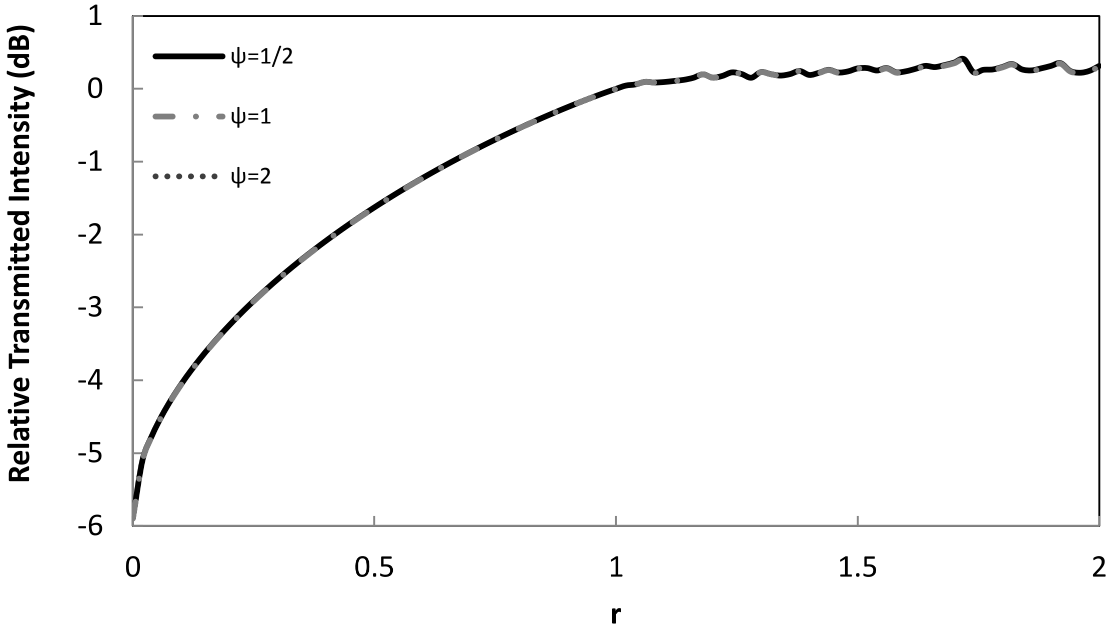

Figure 5.27 . The vibrational field in the 1st panel is excited by a diffuse incident acoustic field. The ratio κ of the wave number in the second panel to that in the first panel is 2. Curves are given for the ratio ψ equals ½, 1 and 2.

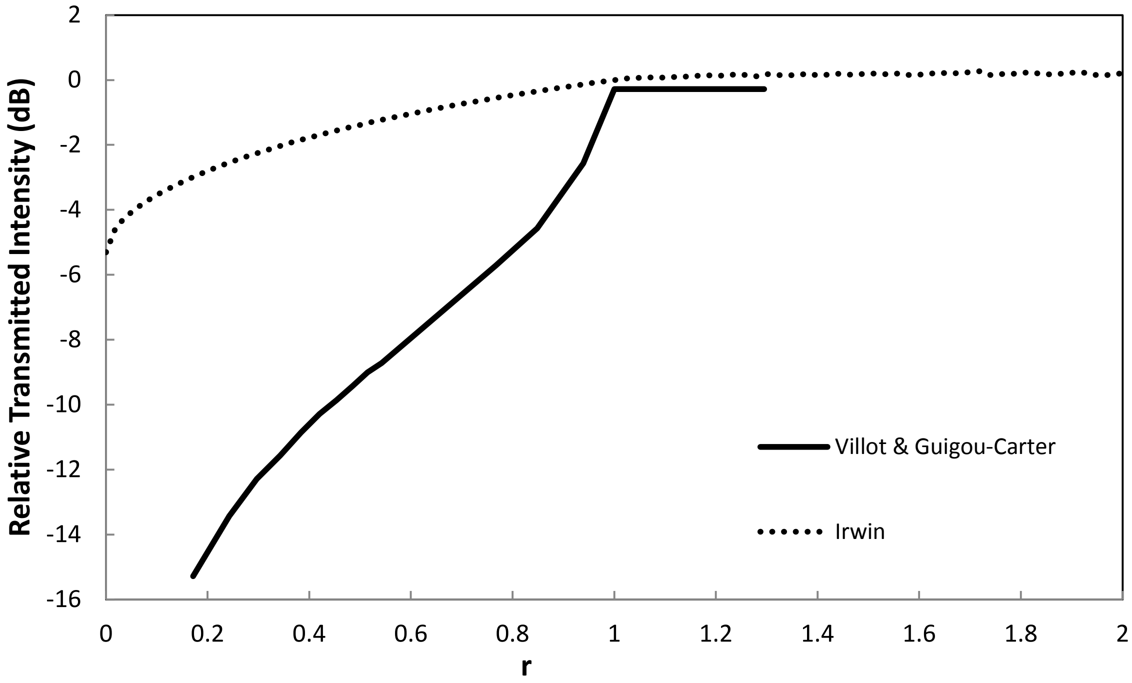

Figure 5.28 The relative intensity transmitted at a pinned joint between two panels when the two panels have the same material properties. The first panel is excited on one of its sides by a diffuse sound field. The Irwin curve shows the calculations made in this thesis, while the Villot & Guigou-Carter curve shows the calculations made by Villot and Guigou-Carter (2000). The *x*-axis variable, *r* is the ratio of the wave number of the diffuse sound field to the free bending wave number of the two identical panels. The critical frequency occurs when r equals one.

The critical frequency occurs when *r* is equal to one. Above the critical frequency, where *r* is greater than or equal to one, the two curves agree well with each other. Below the critical frequency the relative transmitted intensity is much greater than that calculated by Villot and Guigou-Carter (2000). Hence it is not as obvious as claimed by Villot and Guigou-Carter (2000) that the intensity transmitted by the forced wave can be ignored relative to that transmitted by resonant waves.

The production of figure 5.28 was the main aim of this thesis. The identification of two errors in Villot and Guigou-Carter's (2000) calculations meant that it was important to correct their calculations. This has been achieved in figure 5.28 and large differences between the corrected calculations and Villot and Guigou-Carter's (2000) calculations have been identified below the critical frequency.

From equation (4.41), the bending wave number of plate 1 is

$$k_1^2 = \frac{m_1 \omega^2}{B_1}. \tag{5.20}$$

The airborne wave number is

$$k_a = \frac{\omega}{c}. \tag{5.21}$$

The critical angular frequency of plate 1 ω_{c1} or the critical frequency f_{c1} of plate 1 occurs when

$$k_a = k_1. \tag{5.22}$$

Therefore, after much manipulatiuon it is found that,

$$r = \frac{k_a}{k_1} = \sqrt{\frac{\omega}{\omega_{c1}}} = \sqrt{\frac{f}{f_{c1}}}. \tag{5.23}$$

Thus r for Villot and Guigou-Carter's (2000) figure 7 is calculated by taking the square root of the ratio of the frequency divided by the critical frequency. In Villot and Guigou-Carter's (2000) figure 7, the critical frequency is where their forced curve stops decreasing with increasing frequency and becomes almost constant above 3.4 kHz.

It should also be noted that

$$\kappa = \frac{k_2}{k_1} = \sqrt{\frac{\omega_{c2}}{\omega_{c1}}} = \sqrt{\frac{f_{c2}}{f_{c1}}}. \tag{5.24}$$

Chapter 6 Conclusion

It has been shown in this thesis that the transmission of forced waves at an interface between two media is different from the transmission of freely propagating waves. The Villot and Guigou-Carter (2000) equation for the amplitude of the transmitted bending wave at a pinned junction between two panels due to a forced incident wave is correct. However, their expression for the transmitted bending wave intensity is not correct. This is because it is not possible to define a transmission factor for forced incident waves as Villot and Guigou-Carter (2000) have attempted to do.

There appear to be two errors in Villot and Guigou-Carter's (2000) paper. The first error is because it is not possible to calculate the energy incident on a junction by just considering the incident wave and ignoring the reflected wave. The interaction of the incident and reflected waves must be considered. This is because the cross terms between the transverse velocity of one of the waves with the transverse force of the other wave is not zero. This is different from the case when the incident waves are freely propagating and these cross terms are zero.

The second error is due to the weighting over the direction of the incident wave used by Villot and Guigou-Carter (2000). This is incorrect because it has as a $\cos\theta$ term that should not be there. The $\cos\theta$ term comes from the differential of the forced wave number in the panel, which appears in their integrals.

Because of these two errors, Villot and Guigou-Carter's (2000) numerically calculated graph of the transmission loss is incorrect. It shows that the attenuation of the forced wave can be 16dB or more than the attenuation of the freely propagating wave. The calculations in this thesis show that the maximum extra attenuation for the same situation is less than 6dB.

Because of the complex nature of the problem being considered in this thesis, initially the situation of forced acoustical waves propagating from a half infinite fluid medium to another half infinite fluid medium was considered. The normally incident case was considered first. Then the oblique incidence case was considered. Finally the diffuse field incidence case was considered. If the wave speed in the two media is the same, a very simple formula for the ratio of the energy transmitted by the forced diffuse field waves to that transmitted by the freely propagating waves was calculated. Using analytical integration, this energy ratio was found to be (equation (2.180) and (2.188)):

$$\frac{1+r}{2} \qquad r \leq 1, \tag{6.1}$$

and

$$\frac{1}{2}\left(1+\frac{1}{r}\right) \qquad r \geq 1. \tag{6.2}$$

In these formulae, r is the ratio of the wave number of the forced incident acoustical wave to the wave number of a freely propagating wave.

When the wave speed was different (in the two different infinite half media), it was necessary to use numerical integration over the angle of incidence to calculate the diffuse field transmitted energy. This thesis provides graphs showing the ratio of the transmitted intensity for a forced incident wave to that for a freely propagating incident wave.

Then, the case of two half infinite plates connected by a pinned joint was considered. The pinned joint was considered because this is a reasonable approximation for two half infinite plates connected at right angles at low frequencies. Because, for this pinned joint case, the angle between the plates makes no difference, this thesis considers the case when the plates are in the same plane. Again because of the complexity, normal incidence was considered first, followed by the oblique incidence case. Two different diffuse field cases were considered. Firstly, calculations were made for the case of a single forced wave number diffuse incident bending wave field in the first plate. In the second case, the diffuse incident bending wave field in the first plate is excited by a diffuse airborne sound field on one side of the first plate. The forced wave number in the first plate depends on the angle of incidence of the forcing acoustic wave to the normal to the plate.

For the case when the wave in the first plate is excited by a diffuse airborne sound wave as stated above, the transmission of the forced wave can be up to 6dB less than the transmission of the freely propagating wave. Thus the assumption that is often made, when predicting flanking transmission using the methods in the EN12354 standard, that the energy transmitted by the forced wave can be ignored, is not necessarily correct. This is because the extra attenuation of the forced bending wave compared to that of the freely propagating incident waves is not as large as that predicted by Villot and Guigou-Carter (2000). Of course, in many cases the energy of the forced bending waves will be much less than the energy of the freely propagating waves, but that issue is not considered in this thesis.

The research conducted in this thesis could be extended by considering junction types other than pinned junctions. Unfortunately the equations will become even more complicated than those presented in this thesis.

Appendix 1

Matlab code

```
function z = squareroot( x, r, alpha)
ralpha=r.*alpha;
r2alpha2=ralpha.*ralpha;
z=sqrt(1-r2alpha2.*(1-x.*x));
end
```

This function calculates the squareroot terms used in the integral function

```
function y = Integral(r,alpha,beta)
ralpha=r.*alpha;
if ralpha<=1
    lower=0;
else
    lower=sqrt(1-1./(ralpha.*ralpha));
end
y=quadgk(@p,lower,1);
    function z=p(x)
        temp1=squareroot(x,r,1);
        temp2=squareroot(x,r,alpha);
        z=abs(r.*x+temp1).^2.*real(temp2);
        z=z./abs(temp1+temp2.*beta).^2;
    end
end
```

This function calculates the relative transmitted intensity for a diffuse incident sound field in the acoustic case as a function of the ratio *r* of the forced wave number to the freely propagating wave number and the ratios alpha and beta. Alpha is the wave number in medium one divided by the wave number in medium two. Beta is the impedance of medium one divided by the impedance of medium two.

```
function r = transmit(alpha,beta)
lower=0;
upper=2;
number=101;
step=(upper-lower)./(number-1);
r=zeros(number,3);
r(1:number,1)=lower:step:upper;
for m=1:number
    r(m,2)=Integral(r(m,1),alpha,beta);
    if r(m,1)<=1
        r(m,3)=(1+r(m,1))./2;
    else
        r(m,3)=(1+1./r(m,1))./2;
    end
end
r(1:number,2)=r(1:number,2)./Integral(1,alpha,beta);
plot(r(1:number,1),r(1:number,2),r(1:number,1),r(1:number,3))
end
```

This function calculates and graphs the output of the integral function at 101 equally spaced values of *r* from 0 to 2. The values are normalized by the value of the integral function for *r* equals 1. The *r* is equal to the forced wave number divided by the wave number in medium one. The values are calculated for the given values of alpha and beta. Alpha is the wave number in medium one divided by the wave number in medium two. Beta is the impedance of medium one divided by the impedance of medium two.

```
function t = tau ( psi, kappa, chia, theta)
sin2=sin(theta).^2;
costheta=sqrt(1-sin2);
chia2=chia.*chia;
chia2sin2=chia2.*sin2;
kappa2=kappa.*kappa;
x=chia2sin2./kappa2;
y1=sqrt(1-chia2sin2);
y2=sqrt(1+chia2sin2);
y3=sqrt(1-x);
y4=sqrt(1+x);
t=1i.*chia.*costheta+y2+(1i.*y1-y2).*(1+chia2)./2;
b=psi.*(1i.*y1-y2)+kappa.*(1i.*y3-y4);
t=t./b;
t=abs(t).^2.*real(y3);
end
```

This function calculates the relative transmitted intensity for the case of a pinned junction between two panels for a wave incident at an angle of theta to the normal to the junction. The calculation is performed for the given values of psi, kappa and chia. Where psi is the ratio of the bending stiffness of plate two multiplied by the square of the wave number in plate two, to the bending stiffness of plate one by the square of the wave number of plate one. The kappa term is the wave number in plate two divided by the wave number in plate one. The term chia is the wave number of a forced incident wave divided by the wave number of a freely propagating wave number in plate one.

```
function chia = panel(psi, kappa, theta)
lower=0;
upper=2;
number=101;
step=(upper-lower)./(number-1);
chia=zeros(number,2);
chia(1:number,1)=lower:step:upper;
for m=1:number
    chia(m,2)=tau ( psi, kappa, chia(m,1), theta);
end
chia(1:number,2)=chia(1:number,2)./tau ( psi, kappa, 1,
theta);
plot(chia(1:number,1),chia(1:number,2))
end
```

This function graphs the normalized output values of tau as a function of chia. The term chia is the wave number of a forced incident wave divided by the wave number of a freely propagating wave number in plate one. The values are normalized by dividing by the output of tau for the freely propagating case when chia equals 1.

```
function y = inttau(psi, kappa, chia)
[m,n]=size(chia);
 for i=1:m
  for j=1:n
   if kappa>=chia(i,j);
    upper=pi./2;
   else
    upper=asin(kappa./chia(i,j));
   end
   y=quadgk(@p,0,upper);
  end
 end
 function z=p(theta)
  z=tau ( psi, kappa, chia(i,j), theta);
 end
end
```

This function calculates the relative transmitted intensity for the case of a diffuse bending wave field in the first panel by integrating tau over the angle of incidence.

```
function chia = intpanel(psi, kappa)
lower=0;
upper=2;
number=101;
step=(upper-lower)./(number-1);
chia=zeros(number,2);
chia(1:number,1)=lower:step:upper;
for m=1:number
    chia(m,2)=inttau(psi, kappa, chia(m,1));
end
chia(1:number,2)=10*log10(chia(1:number,2)./inttau(psi, kappa,
1));
plot(chia(1:number,1),chia(1:number,2))
end
```

This function graphs the normalized output of inttau as a function of chia for 101 equally spaced values from 0 to 2. The normalization is performed by dividing by the value of intau for the freely propagating case when chia equals one. The term chia is the wave number of a forced incident wave divided by the wave number of a freely propagating wave number in plate one.

```
function y = dblint( psi, kappa, r, neta )
a=quadgk(@p,0,pi/2);
    function z=p(phi)
        sinphi=sin(phi);
        chia=r.*sinphi;
        temp=(chia.^4-1).^2+neta.^2.*chia.^8;
        z=inttau(psi, kappa, chia).*sinphi./temp;
    end
b=quadgk(@q,0,pi/2);
    function z=q(phi)
        sinphi=sin(phi);
        chia=r.*sinphi;
        temp=(chia.^4-1).^2+neta.^2.*chia.^8;
        z=(pi./2).*sinphi./temp;
    end
y=a/b;
end
```

This function calculates the relative transmitted intensity for the case when the incident bending wave field is forced by a diffuse field acoustic wave. It performs a double integral (over angle of incidence and azimuthal angle) by integrating the output of inttau. The values are calculated for the given angles of the ratios psi, kappa, *r* and neta (the damping loss factor of the first panel). Where psi is the ratio of the bending stiffness of plate two multiplied by the square of the wave number in plate two, to the bending stiffness of plate one by the square of the wave number of plate one. The kappa term is the wave number in plate two divided by the wave number in plate one. The *r* is equal to the forced wave number divided by the wave number in medium one. The neta term is the in situ damping loss factor of plate one, it has the value 0.003.

```
function f = graphdblint(psi, kappa, neta)
lower=0;
upper=2;
number=101;
step=(upper-lower)./(number-1);
f=zeros(number,2);
f(1:number,1)=lower:step:upper;
for m=1:number
```

```
    f(m,2)=dblint(psi, kappa, sqrt(f(m,1)), neta);
end
f(1:number,2)=10.*log10(f(1:number,2)./dblint(psi, kappa, 1,
neta));
plot(f(1:number,1),f(1:number,2))
end
```

This function normalizes the output of dblint by dividing it by the output of dblint for the case when *r* equals one. The *r* is equal to the forced wave number divided by the wave number in medium one.

References

Brunskog, J., & Chung, H. (2011). "Non-diffuseness of vibration fields in ribbed plates". *Journal of the Acoustical Society of America, 129*, 1336-1343.

CEN. (2000a). "CEN EN12354-1:2000(E)-Building acoustics-Estimation of acoustic performance in buildings from the performance of elements-Part 1: Airborne sound between rooms" (pp. 1-20). Brussels, Belgium: European Committee for Standardization.

CEN. (2000b). "CEN EN12354-2:2000(E)-Building acoustics-Estimation of acoustic performance in buildings from the performance of elements-Part 2: Impact sound insulation between rooms" (pp. 1-20). Brussels, Belgium: European Committee for Standardization.

CEN. (2000c). "CEN EN12354-3:2000(E)-Building acoustics-Estimation of acoustic performance in buildings from the performance of elements-Part 3: Airborne sound insulation against outdoor sound" (pp. 1-20). Brussels, Belgium: European Committee for Standardization.

CEN. (2000d). "CEN EN12354-4:2000(E)-Building acoustics-Estimation of acoustic performance in buildings from the performance of elements-Part 4: Transmission of indoor sound to the outside" (pp. 1-20). Brussels, Belgium: European Committee for Standardization.

CEN. (2000e). "CEN EN12354-5:2000(E)-Building acoustics-Estimation of acoustic performance in buildings from the performance of elements-Part 5: Sound levels due to the service equipment" (pp. 1-20). Brussels, Belgium: European Committiee for Standardization.

CEN. (2000f). "CEN EN12354-6:2000(E)-Building acoustics-Estimation of acoustic performance in buildings from the performance of elements-Part 6: Sound absoption in enclosed spaces" (pp. 1-20). Brussels, Belgium: European Committee for Standardization.

Cremer, L, Heckl, M, & Petersson, B.A.T. (2005). *Structure-Borne Sound*. Berlin, Germany: Springer.

Crispin, C., Ingelaere, B., Van Damme, M., & Wuyts, D. (2006). "The vibration reduction index Kij: Laboratory measurements for rigid junctions and for junctions with flexible interlayers,". *Journal of Building Acoustics, 13*, 99-112.

CSTC. (2008). Détails de construction pour améliorer l'isolationaux bruits aériens pour les constructions légères: Laboratoire acoustique, Centre Scientifique et Technique de la Construction (CSTC), Limelette, Belgium.

Davy, John. (2009). The directivity of the sound radiation from panels and openings. *Journal of the Acoustical Society of America, 125*, 3795-3805.

Davy, John.L, Mahn, J.P, Guigou-Carter, Catherine, & Villot, Michel. (2012). The prediction of flanking sound transmission below the critical frequency. *Journal of the Acoustical Society of America, 132*(4), 12. doi: 10.1121/1.4746945

Gerretsen, E. (1979). Calculation of the sound transmission between dwellings by partitions and flanking structures. *Applied Acoustics*, 413-433.

Gerretsen, E. (1986). Calculation of Airborne and Impact Sound Insulation Between Dwellings. *Applied Acoustics, 19*, 245-264.

Gerretsen, Eddy. (2007). *Some aspects to improve sound insulation prediction models for lightweight elements.* Paper presented at the Inter-noise 2007

Istanbul, Turkey.

Gradshteyn, I.S, & Ryzhik, I.M. (1980). *Table of Integrals, Series, and Products* (A. Jeffrey Ed. 4th edition ed.). United States of America: Academic Press.

Guigou-Carter, Catherine, Villot, Michel, & Roland, Wetta. (2006). Prediction Method Adapted to Wood Frame Lightweight Constructions. *Journal of Building Acoustics, 13*, 173-188.

ISO. (2005a). "ISO 15712-1:2005(E) Building acoustics-Estimation of acoustic performance of buildings from the performance of elements-Part 1: Airborne sound insulation between rooms" (pp. 1-59). Geneva, Switzerland: International Organization for standardization.

ISO. (2005b). "ISO 15712-2:2005(E) Building acoustics-Estimation of acoustic performance of buildings from the performance of elements-Part 2:Impact sound insulation between rooms" (pp. 30). Geneva, Switzerland: International Organization for standardization.

ISO. (2005c). "ISO 15712-3:2005(E) Building acoustics-Estimation of the acoustic performance of buildings from the performance of elements-Part 3: Airborne sound insulation against outdoor sound" (pp. 1-31). Geneva, Switzerland: International Organization for Standardization.

ISO. (2005d). "ISO 15712-4:2005(E) Building acoustics-Estimation of acoustic performance of buildings from the performance of elements-Part 4: Transmission of indoor sound to the outside (pp. 1-28). Geneva, Switzerland: International organization for standardization

ISO. (2006). "ISO 10848-2:2006(E)-Acoustics-Laboratory measurement of the flanking transmission of airborne and impact sound between adjoining rooms-Part2: Application to light elements when the junction has a substantial influence-Technical Corrigendum 1". Geneva, Switzerland: International Organization for Standardization.

ISO. (2006a). "ISO 10848-1:2006(E)-Acoustics-Laboratory measurement of the flanking transmission of airborne and impact sound between adjoining rooms-Part1:Frame document" (pp. 1-30). Geneva, Switzerland: International Organization for Standardization.

ISO. (2006b). "ISO 10848-2:2006(E)-Acoustics-Laboratory measurement of the flanking transmission of airborne and impact sound between adjoining rooms- Part2: Application to light elements when the junction has a small influence" (pp. 1-16). Geneva, Switzerland: International Organization for Standardization.

Mahn, Jeffrey P. (2008). *Prediction of Flanking Noise Transmission in Lightweight Building Constructions: A Theoretical and Experimental Evaluation of the Application of EN12354-1*. (PhD Research Thesis), University of Canterbury, Christchurch, New Zealand.

Nightingale, T.R.T. (1995). Application of the CEN draft building acoustics prediction model to a lightweight double leaf construction. *Applied Acoustics*, *46*, 265-284.

Nightingale, T.R.T, & Bosmans, I. (2003). Expressions for first-order flanking paths in homogeneous isotropic and lightly damped buildings. *Acta Acoustica*, *89*, 110-122.

Pavasovic, Vladimir. (2006). *The radiation of sound from surfaces at grazing angles of incidence* (Master of Applied Science Research), RMIT University, Melbourne. Retrieved from http://adt.lib.rmit.edu.au/adt/public/adt-VIT20060911.115939

Villot, Michel. (2002). *Modeling flanking transmissions in lightweight constructions*. Paper presented at the Forum Acustica 2002, Seville Spain.

Villot, Michel, & Guigou-Carter, Catherine. (2000). *Contribution of forced and resonant vibration in sound transmission through partitions and in vibration transmission through plate junctions*. Paper presented at the NOVEM 2000. Noise and Vibration, Emerging Methods, Lyon, France.